Complete Solutions Manual

Technical Calculus with Analytic Geometry

FIFTH EDITION

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Chapter 1

Introduction to Analytic Geometry

1.1 The Cartesian Coordinate System

1. Let \((x_2, y_2) = (2, 4)\) and \((x_1, y_1) = (5, 2)\). From the distance formula

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

we get

\[ d = \sqrt{(2 - 5)^2 + (4 - 2)^2} = \sqrt{(-3)^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}. \]

2. Let \((x_2, y_2) = (-3, 2)\) and \((x_1, y_1) = (5, -4)\). From the distance formula

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

we get

\[ d = \sqrt{(-3 - 5)^2 + (2 - (-4))^2} = \sqrt{(-8)^2 + 6^2} = \sqrt{64 + 36} = 10. \]

3. Let \((x_2, y_2) = (-3, -6)\) and \((x_1, y_1) = (5, -2)\). Then

\[ d = \sqrt{(-3 - 5)^2 + (-6 - (-2))^2} = \sqrt{(-8)^2 + (-4)^2} = \sqrt{64 + 16} = \sqrt{16 \cdot 5} = 4\sqrt{5}. \]

4. Let \((x_2, y_2) = (-\sqrt{5}, 2)\) and \((x_1, y_1) = (0, 0)\).

Then \( d = \sqrt{(-\sqrt{5} - 0)^2 + (2 - 0)^2} = \sqrt{5 + 4} = 3 \)

5. Let \((x_2, y_2) = (\sqrt{3}, 4)\) and \((x_1, y_1) = (0, 2)\). Then

\[ d = \sqrt{(\sqrt{3} - 0)^2 + (4 - 2)^2} = \sqrt{3 + 4} = \sqrt{7}. \]

6. \( d = \sqrt{\left(\sqrt{2} - \sqrt{2}\right)^2 + (\sqrt{5} - 0)^2} = \sqrt{0 + 5} = \sqrt{5} \)

7. \( d = \sqrt{[1 - (-1)]^2 + (-\sqrt{2} - 0)^2} = \sqrt{4 + 2} = \sqrt{6} \)

8. \( d = \sqrt{[2 - (-2)]^2 + (7 - 3)^2} = \sqrt{16 + 16} = \sqrt{2 \cdot 16} = 4\sqrt{2} \)

9. \( d = \sqrt{[-9 - (-11)]^2 + (-1 - 1)^2} = \sqrt{24 + (-2)^2} = \sqrt{8} = \sqrt{2 \cdot 4} = 2\sqrt{2} \)

10. Distance from \((0, 0)\) to \((4, 3)\): \( \sqrt{(4 - 0)^2 + (3 - 0)^2} = \sqrt{16 + 9} = 5 \). Distance from \((0, 0)\) to \((6, 0)\): \( \sqrt{(6 - 4)^2 + (0 - 3)^2} = \sqrt{4 + 9} = \sqrt{13} \).

Perimeter = \( 5 + 6 + \sqrt{13} = 11 + \sqrt{13} \).
11b. \( x/y \) is negative whenever \( x \) and \( y \) have opposite signs: quadrants II and IV.

12. If \( x_2 > x_1 \), then \( x_2 - x_1 = P_1P_3 = |x_2 - x_1| \). If \( x_2 < x_1 \), then \( x_2 - x_1 = -P_1P_3 \) and \( P_1P_3 = |P_1P_3| = |x_2 - x_1| \).

13a. Any point on the \( y \)-axis has coordinates of the form \((0, y)\).

14. Distance from \((11, 2)\) to origin: \( \sqrt{(11 - 0)^2 + (2 - 0)^2} = \sqrt{125} \)
   Distance from \((-5, 10)\) to origin: \( \sqrt{(-5 - 0)^2 + (10 - 0)^2} = \sqrt{125} \)
   Distance from \((-1, -11)\) to origin: \( \sqrt{(-1 - 0)^2 + (-11 - 0)^2} = \sqrt{122} \)
   Answer: \((-1, -11)\)

15. Let \( A = (-2, -5) \), \( B = (-4, 1) \) and \( C = (5, 4) \); then \( AB = \sqrt{(-2 - (-4))^2 + (-5 - 1)^2} = \sqrt{40}, AC = \sqrt{(-2 - 5)^2 + (-5 - 4)^2} = \sqrt{130}, \) and \( BC = \sqrt{(-4 - 5)^2 + (1 - 4)^2} = \sqrt{90} \).
   Since \( AB^2 + BC^2 = AC^2 \), the triangle must be a right triangle.

16. Let \( A = (-1, -1), B = (-2, 3) \), and \( C = (6, 5) \). After calculating \( AB = \sqrt{17}, BC = \sqrt{68} \), and \( AC = \sqrt{85} \), we observe that \( AB^2 + BC^2 = AC^2 \).

17. The points \((12, 0), (-4, 8)\) and \((-1, -13)\) are all \(5\sqrt{5}\) units from \((1, -2)\).

18. Distance from \((-2, 10)\) to \((3, -2)\): 13. Distance from \((15, 3)\) to \((3, -2)\): 13.

19. Distance from \((-1, -1)\) to \((2, 8)\):
   \[ \sqrt{(-1 - 2)^2 + (-1 - 8)^2} = \sqrt{9 + 81} = \sqrt{90} = \sqrt{9 \cdot 10} = 3\sqrt{10} \]
   Distance from \((2, 8)\) to \((5, 17)\):
   \[ \sqrt{(5 - 2)^2 + (17 - 8)^2} = \sqrt{90} = 3\sqrt{10} \]
   Distance from \((-1, -1)\) to \((5, 17)\):
   \[ \sqrt{6^2 + 18^2} = \sqrt{360} = 6\sqrt{10} \]
   Total distance \(6\sqrt{10} = 3\sqrt{10} + 3\sqrt{10}\), the sum of the other two distances.

20. Distance from \((x, y)\) to \((-1, 2)\):
   \[ d = \sqrt{(x + 1)^2 + (y - 2)^2} = 3 \]
   \[ (x + 1)^2 + (y - 2)^2 = (3)^2 \] squaring both sides
   \[ x^2 + 2x + 1 + y^2 - 4y + 4 = 9 \]
   \[ x^2 + y^2 + 2x - 4y = 4 \]

21. Distance from \((x, y)\) to \(y\)-axis: \(x\) units
   Distance from \((x, y)\) to \((2, 0)\): \( \sqrt{(x - 2)^2 + (y - 0)^2} = \sqrt{(x - 2)^2 + y^2} \)
   By assumption,
   \[ \sqrt{(x - 2)^2 + y^2} = x \]
   \[ (x - 2)^2 + y^2 = x^2 \] squaring both sides
   \[ x^2 - 4x + 4 + y^2 = x^2 \]
   \[ y^2 - 4x + 4 = 0 \]
22. Let \((x_1, y_1) = (-3, -5)\) and \((x_2, y_2) = (-1, 7)\). Then from the midpoint formula
\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + (-1)}{2}, \frac{-5 + 7}{2}\right) = (-2, 1).
\]

23. Let \((x_1, y_1) = (-2, 6)\) and \((x_2, y_2) = (2, -4)\). Then from the midpoint formula
\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]
we get
\[
\left(\frac{-2 + 2}{2}, \frac{6 + (-4)}{2}\right) = (0, 1).
\]

24. \[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + (-2)}{2}, \frac{5 + 9}{2}\right) = \left(-\frac{5}{2}, 7\right)
\]

25. Let \((x_1, y_1) = (5, 0)\) and \((x_2, y_2) = (9, 4)\). Then from the midpoint formula
\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]
we get
\[
\left(\frac{5 + 9}{2}, \frac{0 + 4}{2}\right) = (7, 2).
\]

26. \[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4 + (-1)}{2}, \frac{3 + (-7)}{2}\right) = \left(-\frac{5}{2}, -2\right)
\]

27. The center is the midpoint: \[
\left(\frac{-2 + 6}{2}, \frac{-1 + 11}{2}\right) = (2, 5).
\]

28. Midpoint of given line segment: (2, 6). Midpoint of line segment from (2, 6) to (-2, 4): (0, 5).

1.2 The Slope

1. Let \((x_2, y_2) = (1, 7)\) and \((x_1, y_1) = (2, 6)\). Then, by formula (1.4),
\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]
we get
\[
m = \frac{7 - 6}{1 - 2} = \frac{1}{-1} = -1.
\]

2. Let \((x_2, y_2) = (-3, -10)\) and \((x_1, y_1) = (-5, 2)\). Then by formula (1.4)
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - 2}{-3 - (-5)} = \frac{-12}{2} = -6.
\]

3. Let \((x_1, y_1) = (0, 2)\) and \((x_2, y_2) = (-4, -4)\). Then
\[
m = \frac{-4 - 2}{-4 - 0} = \frac{-6}{-4} = \frac{3}{2}
\]

4. Let \((x_2, y_2) = (6, -3)\) and \((x_1, y_1) = (4, 0)\). Then \[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{6 - 4} = \frac{-3}{2} = \frac{-3}{2}.
\]

5. Let \((x_2, y_2) = (7, 8)\) and \((x_1, y_1) = (-3, -4)\). Then
\[
m = \frac{8 - (-4)}{7 - (-3)} = \frac{8 + 4}{7 + 3} = \frac{12}{10} = \frac{6}{5}.
\]
6. \( m = \frac{-4 - 0}{8 - 0} = -\frac{1}{2} \)

7. \( m = \frac{43 - (-1)}{-1 - 1} = \frac{44}{-2} = -22 \)

8. \( m = \frac{31 - 1}{0 - (-1)} = 30 \)

9. \( m = \frac{-5 - 4}{3 - 3} = \frac{-9}{0} \) (undefined)

10. \( m = \frac{6 - 6}{5 - (-3)} = \frac{0}{8} = 0 \)

11. \( m = \frac{-3 - (-3)}{9 - 5} = \frac{0}{4} = 0 \)

12. \( m = \frac{3 - (-2)}{4 - 4} = \frac{5}{0} \) (undefined)

13. \( m = \frac{3 - 2}{12 - (-2)} = \frac{1}{14} \)

14. (a) \( \tan 0^\circ = 0 \); (b) \( \tan 30^\circ = \frac{\sqrt{3}}{3} \); (c) \( \tan 150^\circ = -\frac{\sqrt{3}}{3} \); (d) \( \tan 90^\circ \) is undefined; (e) \( \tan 45^\circ = 1 \); (f) \( \tan 135^\circ = -1 \)

15. See answer section of book.

16. Slope of \( AB = \frac{0 - 2}{-2 - (-1)} = \frac{-2}{-1} = 2 \); of \( BC = \frac{5}{3} \); of \( AC = \frac{-3}{4} \).

17. Slope of given line is \( \frac{1 - (-5)}{-7 - 6} = \frac{6}{-13} = -\frac{6}{13} \). Slope of perpendicular is given by the negative reciprocal and is therefore \( \frac{-1}{-6/13} = \frac{13}{6} \).

18. Slope of line through \((-4, 6)\) and \((-1, -3)\): \( \frac{-3 - 6}{-1 - (-4)} = -3 \).
Slope of line through \((-4, 6)\) and \((1, -9)\): \( \frac{-9 - 6}{1 - (-4)} = -3 \).
Since the lines have the same slope and pass through \((-4, 6)\), they must coincide.

19. Slope of line through \((-4, 6)\) and \((6, 10)\): \( \frac{6 - 10}{6 - 4} = \frac{-4}{2} = -2 \).
Slope of line through \((6, 10)\) and \((10, 0)\): \( \frac{10 - 0}{6 - 10} = \frac{10}{-4} = -\frac{5}{2} \).
Since the slopes are negative reciprocals, the lines are perpendicular.

20. Slope of line through \((-4, 2)\) and \((-1, 8)\): 2;
Slope of line through \((9, 4)\) and \((6, -2)\): 2;
Slope of line through \((-1, 8)\) and \((9, 4)\): \(-\frac{2}{5}\);
Slope of line through \((-4, 2)\) and \((6, -2)\): \(-\frac{2}{5}\);

21. Slope of line through \((0, -3)\) and \((-2, 3)\): \( \frac{-3 - 3}{0 - (-2)} = \frac{-6}{2} = -3 \)
Slope of line through \((7, 6)\) and \((9, 0)\): \( \frac{6 - 0}{7 - 9} = \frac{6}{-2} = -3 \)
Slope of line through \((-2, 3)\) and \((7, 6)\): \( \frac{3 - 6}{-2 - 7} = \frac{-3}{-9} = \frac{1}{3} \)
Slope of line through \((0, -3)\) and \((9, 0)\): \( \frac{-3 - 0}{0 - 9} = \frac{1}{3} \)
Since $-3$ and $\frac{1}{3}$ are negative reciprocals, adjacent sides are perpendicular and opposite sides are parallel.

22. Midpoint of line segment: \( \left( \frac{-2 + 8}{2}, \frac{-4 + 8}{2} \right) = (3, 2) \). Point: \((6, -4), m = \frac{-4 - 2}{6 - 3} = -2\)

23. Midpoint: \( \left( \frac{-3 + 9}{2}, \frac{-2 + 0}{2} \right) = (3, -1) \)

Slope of line through \((5, 6)\) and \((3, -1)\): \( \frac{6 - (-1)}{5 - 3} = \frac{7}{2} \)

24. Let \((x, y)\) be the other end of the diameter. Since the center is the midpoint, we get \( \frac{x}{2} + 4 = 1 \) and \( \frac{y - 3}{2} = 2 \); solving, \( x = -2, y = 7 \).

25. \( \tan \theta = \frac{\text{rise}}{\text{run}} = \frac{10.0 \text{ ft}}{160 \text{ ft}} = 0.0625 \)

26. \( \tan \theta = \frac{\text{rise}}{\text{run}} = \frac{2.5 \text{ m}}{19.0 \text{ m}} \approx 0.1316; \theta = 7.5^\circ \)

27. Slope of line through \((-1, -1)\) and \((3, -5)\): \( \frac{-1 - (-5)}{-1 - 3} = \frac{4}{-4} = -1 \)

Slope of line through \((x, 2)\) and \((4, -6)\): \( \frac{2 + 6}{x - 4} = \frac{8}{x - 4} \)

Since the two slopes must be equal, we have:
\[
\frac{8}{x - 4} = -1
\]
\[
8 = -x + 4 \quad \text{multiplying both sides by } x - 4
\]
\[
x = -4
\]

28. Slope of line segment \((2, -1)\) to \((-3, 2)\): \( \frac{-3}{5} \); Slope of other line segment: \( \frac{-2 + 7}{x - 4} \); so \( \frac{5}{x - 4} = \frac{5}{3} \) (negative reciprocals); solving, \( x = 7 \).

### 1.3 The Straight Line

1. Since \((x_1, y_1) = (-7, 2)\) and \(m = 1/2\), we get

\[
y - 2 = \frac{1}{2}(x + 7) \quad y - y_1 = m(x - x_1)
\]

\[
2y - 4 = x + 7 \quad \text{clearing fractions}
\]

\[
x - 2y + 11 = 0
\]

2. Since \((x_1, y_1) = (0, 3)\) and \(m = -4\), we get

\[
y - 3 = -4(x - 0) \quad y - y_1 = m(x - x_1)
\]

\[
4x + y - 3 = 0
\]

3. \( y - y_1 = m(x - x_1) \)

\[
y + 4 = 3(x - 3) \quad (x_1, y_1) = (3, -4); \ m = 3
\]

\[
y + 4 = 3x - 9
\]

\[
3x - y - 13 = 0
\]

4. By \((1,8), y = 2\).
5. \[ y - y_1 = m(x - x_1) \]
   \[ y - 0 = -\frac{1}{3}(x - 0) \quad (x_1, y_1) = (0, 0); \quad m = -1/3 \]
   \[ 3y = -x \]
   \[ x + 3y = 0 \]

6. By (1.9), \( x = -3 \).

7. The line \( y = 1 = 0x + 1 \) has slope 0.
   \[ y - y_1 = m(x - x_1) \]
   \[ y - 0 = 0(x + 4) \quad (x_1, y_1) = (-4, 0); \quad m = 0 \]
   \[ y = 0 \quad \text{x-axis} \]

8. First determine the slope: \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{-3 - 7} = \frac{2}{5} \)
   Let \((x_1, y_1) = (-3, 2)\) then
   \[ y - 2 = \frac{2}{5}(x + 3) \]
   \[ 5y - 10 = 2x + 6 \quad \text{multiplying by 5} \]
   \[ 2x - 5y + 16 = 0 \]

9. First determine the slope using \[ m = \frac{y_2 - y_1}{x_2 - x_1} \text{ to get } m = \frac{4 - (-6)}{-3 - 3} = \frac{10}{-6} = -\frac{5}{3}. \]
   Then let \((x_1, y_1) = (-3, 4)\) to get
   \[ y - 4 = -\frac{5}{3}(x + 3) \]
   \[ 3y - 12 = -5x - 15 \quad \text{multiplying by 3} \]
   \[ 5x + 3y + 3 = 0 \]

10. \[ m = \frac{6 - 3}{-4 - 2} = -\frac{1}{2} \]
    \[ y - 3 = -\frac{1}{2}(x - 2) \quad (x_1, y_1) = (2, 3) \]
    \[ 2y - 6 = -x + 2 \quad \text{multiplying by 2} \]
    \[ x + 2y - 8 = 0 \]

11. \[ m = \frac{-4 - 0}{9 - 5} = -1 \]
    \[ y - 0 = -1(x - 5) \quad \text{choosing } (x_1, y_1) = (5, 0) \]
    \[ x + y - 5 = 0 \]

12. \[ m = \frac{5 - 7}{3 - 1} = \frac{1}{2} \]
    \[ y - 5 = \frac{1}{2}(x + 3) \quad (x_1, y_1) = (-3, 5) \]
    \[ 2y - 10 = x + 3 \quad \text{multiplying by 2} \]
    \[ x - 2y + 13 = 0 \]

13. \[ 6x + 2y = 5 \]
    \[ 2y = -6x + 5 \]
    \[ y = -3x + \frac{5}{2} \quad y = mx + b \]
    \[ m = -3, \quad y\text{-intercept } = \frac{5}{2}; \text{ see graph in answer section of book}. \]

14. Solving for \( y \), we get \( y = x - 1 \). Slope:1, \( y\)-intercept :\(-1\)

15. Since \( 2x = 3y \), \( y = \frac{2}{3}x \). From the form \( y = mx + b \), \( m = \frac{2}{3} \) and \( b = 0 \). The line passes through the origin and has slope \( \frac{2}{3} \). See graph in answer section of book.

16. From \( y = -4x + 12 \), we get \( m = -4 \) and \( b = 12 \).
17. \(2y - 7 = 0\)
   \[y = 0x + \frac{7}{2}\]
   \[y = mx + b\]
   \[m = 0, \text{ y-intercept} = \frac{7}{2}; \text{ see graph in answer section of book.}\]

18. Solving for \(y\):
   \[y = \frac{1}{4}x - \frac{3}{2}\]
   slope: \(\frac{1}{4}\); y-intercept: \(-\frac{3}{2}\)

19. \(2x - 3y = 1\)
   \[4x - 6y + 3 = 0\]
   \[-3y = -2x + 1\]
   \[-6y = -4x - 3\]
   \[y = \frac{2}{3}x - \frac{1}{3}\]
   \[y = \frac{4}{9}x + \frac{3}{9}\]
   From the form \(y = mx + b\), \(m = \frac{2}{3}\) in both cases, so that the lines are parallel.

20. \(2x + 4y + 3 = 0\)
   \[y - 2x = 2\]
   \[y = -\frac{1}{2}x - \frac{3}{4}\]
   slope: \(-\frac{1}{2}\)
   Answer: The lines are perpendicular.

21. \(3x - 4y = 1\)
   \[3y - 4x = 3\]
   \[-4y = -3x + 1\]
   \[3y = 4x + 3\]
   \[y = \frac{3}{4}x - \frac{4}{5}\]
   \[y = \frac{9}{15}x + \frac{4}{15}\]
   The lines are neither parallel nor perpendicular.

22. \(7x - 10y = 6\)
   \[y - 4 = 0\]
   \[y = \frac{7}{10}x - \frac{3}{5}\]
   slope: \(\frac{7}{10}\)
   Answer: neither

23. \(x + 3y = 5\)
   \[y - 3x - 2 = 0\]
   \[3y = -x + 5\]
   \[y = -\frac{1}{3}x + \frac{5}{3}\]
   The slopes are \(-\frac{1}{3}\) and 3, respectively. Since the slopes are negative reciprocals, the lines are perpendicular.

24. \(2x + 5y = 2\)
   \[6x + 15y = 1\]
   \[y = -\frac{2}{5}x + \frac{2}{5}\]
   \[y = -\frac{2}{5}x + \frac{1}{15}\]
   Slope is \(-\frac{2}{5}\) in each case; the lines are parallel.

25. \(3x - 5y = 6\)
   \[9x - 15y = 4\]
   \[5y = -3x + 6\]
   \[-15y = -9x + 4\]
   \[y = \frac{3}{5}x - \frac{6}{5}\]
   \[y = \frac{-9}{15}x + \frac{4}{15}\]
   From the form \(y = mx + b\), the slope \(m\) is \(\frac{3}{5}\) in both cases; so the lines are parallel.

26. \(4y - 3x + 6 = 0\)
   \[6x - 8y + 1 = 0\]
   \[4y = 3x - 6\]
   \[-8y = -6x - 1\]
   \[y = \frac{3}{4}x - \frac{3}{2}\]
   \[y = -\frac{6}{8}x + \frac{1}{8}\]
   \[y = \frac{3}{4}x + \frac{1}{8}\]
   slope: \(\frac{3}{4}\)
   Answer: the lines are parallel.
27. \[ 2y - 3x = 6 \quad 6y + 4x = 5 \]
\[ 2y = 3x + 6 \quad 6y = -4x + 5 \]
\[ y = \frac{3}{2}x + 3 \quad y = -\frac{4}{6}x + \frac{5}{6} \]
\[ y = -\frac{2}{3}x - \frac{5}{6} \]

The respective slopes are \( \frac{3}{2} \) and \( -\frac{2}{3} \). Since the slopes are negative reciprocals, the lines are perpendicular.

28. \[ x - 4y - 2 = 0 \quad 2y + 8x - 7 = 0 \]
\[ -4y = -x + 2 \quad 2y = -8x + 7 \]
\[ y = \frac{1}{4}x - \frac{1}{2} \quad y = -4x + \frac{7}{2} \]

The respective slopes are \( \frac{1}{4} \) and \( -4 \). Since the slopes are negative reciprocals, the lines are perpendicular.

29. \[ 3y - 2x - 12 = 0 \quad 2x + 3y - 4 = 0 \]
\[ 3y = 2x + 12 \quad 3y = -2x + 4 \]
\[ y = \frac{2}{3}x + 4 \quad y = -\frac{2}{3}x + \frac{4}{3} \]

The lines are neither parallel nor perpendicular.

30. \[ 3x + 4y - 4 = 0 \quad 6x - 8y - 3 = 0 \]
\[ 4y = -3x + 4 \quad -8y = -6x + 3 \]
\[ y = -\frac{3}{4}x + 1 \quad y = \frac{3}{4}x - \frac{3}{8} \]

The lines are neither parallel nor perpendicular.

31. \( 3x + 4y = 5 \) (given line)
\[ y = -\frac{3}{4}x + \frac{5}{4} \] (slope \( = -\frac{3}{4} \))
\[ y - y_1 = m(x - x_1) \] (point-slope form)
\[ y - 1 = -\frac{3}{4}(x + 2) \] (point: \((-2, 1)\))
\[ 4y - 4 = -3x - 6 \]
\[ 3x + 4y + 2 = 0 \]

32. The given line can be written \( y = -\frac{3}{4}x + \frac{5}{4} \). So the slope is \( -\frac{3}{4} \). Slope of perpendicular: \( \frac{4}{3} \).
\[ y - 1 = \frac{4}{3}(x + 2) \quad y - y_1 = m(x - x_1) \]
\[ 3y - 3 = 4x + 8 \]
\[ 4x - 3y + 11 = 0 \]

33. To find the coordinates of the point of intersection, solve the equations simultaneously:
\[ 2x - 4y = 1 \]
\[ 3x + 4y = 4 \]
\[ 5x = 5 \quad \text{adding} \]
\[ x = 1 \]

From the second equation, \( 3(1) + 4y = 4 \), and \( y = \frac{1}{4} \). So the point of intersection is \((1, \frac{1}{4})\).
From the equation \( 5x + 7y + 3 = 0 \), we get
\[ 7y = -5x - 3 \]
\[ y = -\frac{5}{7}x - \frac{3}{7} \] (slope \( = -\frac{5}{7} \))
Thus \((x_1, y_1) = (1, \frac{1}{4})\) and \(m = -\frac{5}{7}\). The desired line is \(y - \frac{1}{4} = -\frac{5}{7}(x - 1)\). To clear fractions, we multiply both sides by 28:

\[
\begin{align*}
28y - 7 & = -20(x - 1) \\
28y - 7 & = -20x + 20 \\
20x + 28y - 27 & = 0
\end{align*}
\]

34. The first two lines are perpendicular: \(y = \frac{1}{3}x + 1\) and \(y = -3x - \frac{25}{4}\).

35. See graph in answer section of book.

36. \(F = 3x\), slope = 3, passing through the origin.

37. From \(F = kx\), we get \(3 = k \cdot \frac{1}{2}\). Thus \(k = 6\) and \(F = 6x\).

38. \(y\)-intercept: initial value; \(t\)-intercept: the year the value becomes zero

39. \[
\begin{align*}
F & = mC + b \\
212 & = m(100) + b & F = 212, C = 100 \\
32 & = m(0) + b & F = 32, C = 0 \\
\hline
b & = 32 & \text{second equation} \\
212 & = m(100) + 32 & \text{substituting into first equation}
\end{align*}
\]

\[
m = \frac{180}{100} = \frac{9}{5}
\]

Solution: \(F = \frac{9}{5}C + 32\)

40. \(C = 1800 + 500t\)

41. \[
\begin{align*}
R & = aT + b \\
51 & = a \cdot 100 + b & R = 51, T = 100 \\
54 & = a \cdot 400 + b & R = 54, T = 400 \\
-3 & = -300a & \text{subtracting} \\
a & = \frac{-3}{-300} = 0.01
\end{align*}
\]

From the first equation, \(51 = a \cdot 100 + b\), we get

\[
\begin{align*}
51 & = (0.01)(100) + b & (a = 0.01) \\
b & = 50
\end{align*}
\]

So the formula \(R = aT + b\) becomes \(R = 0.01T + 50\).

42. \(P = kx\); let \(P = 187.2\) lb and \(x = 3.0\) ft. Then

\[
187.2 = k(3.0) \quad \text{and} \quad k = \frac{187.2}{3.0} = 62.4
\]

So the relationship is \(P = 62.4x\).
1.4 Curve Sketching

In the answers below, the intercepts are given first, followed by symmetry, asymptotes, and extent.

2. \( y = 2, \ x = \frac{1}{2}; \) none; none; all \( x \)

\[ \text{Intercepts. If } x = 0, \text{ then } y = -9. \text{ If } y = 0, \text{ then} \]
\[ 0 = x^2 - 9 \]
\[ x^2 = 9 \quad \text{solving for } x \]
\[ x = \pm 3 \quad x = 3 \text{ and } x = -3. \]
Symmetry. If \( x \) is replaced by \(-x\), we get \( y = (-x)^2 - 9 \), which reduces to the given equation \( y = x^2 - 9 \). The graph is therefore symmetric with respect to the \( y \)-axis. There is no other type of symmetry.
Asymptotes. Since the equation is not in the form of a quotient with a variable in the denominator, there are no asymptotes.
Extent. \( y \) is defined for all \( x \).
Graph.

4. \( y = 1; \ y\)-axis; none; all \( x \)

\[ \text{Intercepts. If } x = 0, \text{ then } y = 1. \text{ If } y = 0, \text{ then} \]
\[ 0 = 1 - x^2 \]
\[ x^2 = 1 \quad \text{solving for } x \]
\[ x = \pm 1 \quad x = 1 \text{ and } x = -1. \]
Symmetry. If $x$ is replaced by $-x$, we get $y = 1 - (-x)^2$, which reduces to the given equation $y = 1 - x^2$. The graph is therefore symmetric with respect to the $y$-axis. There is no other type of symmetry.

Asymptotes. Since the equation is not in the form of a quotient with a variable in the denominator, there are no asymptotes.

Extent. $y$ is defined for all $x$.

Graph.

\[ y = 1 - x^2. \]

6. $y = 5, x = \pm \sqrt{5}$; $y$-axis; none; all $x$

7. Intercepts. If $x = 0$, then $y = 0$, and if $y = 0$, then $x = 0$. So the only intercept is the origin.

Symmetry. If we replace $x$ by $-x$, we get $y^2 = -x$, which does not reduce to the given equation. So there is no symmetry with respect to the $y$-axis.

If $y$ is replaced by $-y$, we get $(-y)^2 = x$, which reduces to $y^2 = x$, the given equation. It follows that the graph is symmetric with respect to the $x$-axis.

To check for symmetry with respect to the origin, we replace $x$ by $-x$ and $y$ by $-y$: $(-y)^2 = -x$. The resulting equation, $y^2 = -x$, does not reduce to the given equation. So there is no symmetry with respect to the origin.

Asymptotes. Since the equation is not in the form of a fraction with a variable in the denominator, there are no asymptotes.

Extent. Solving the equation for $y$ in terms of $x$, we get

\[ y = \pm \sqrt{x}. \]

Note that to avoid imaginary values, $x$ cannot be negative. It follows that the extent is $x \geq 0$. 

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Graph.

8. origin; x-axis; none; $x \geq 0$

9. Intercepts. If $x = 0$, then $y = \pm 1$. If $y = 0$, then $x = -1$.

Symmetry. If we replace $x$ by $-x$ we get $y^2 = -x + 1$, which does not reduce to the given equation. So there is no symmetry with respect to the $y$-axis.

If $y$ is replaced by $-y$, we get $(-y)^2 = x + 1$, which reduces to $y^2 = x + 1$, the given equation.

It follows that the graph is symmetric with respect to the $x$-axis.

The graph is not symmetric with respect to the origin.

Asymptotes. Since the equation is not in the form of a fraction with a variable in the denominator, there are no asymptotes.

Extent. Solving the equation for $y$, we get $y = \pm \sqrt{x + 1}$. To avoid imaginary values, we must have $x + 1 \geq 0$ or $x \geq -1$. Therefore the extent is $x \geq -1$.

Graph.

10. $y = \pm \sqrt{2}$; $x = 2$; x-axis; none; $x \leq 2$
11. **Intercepts.** If \( x = 0 \), then \( y = (0 - 3)(0 + 5) = -15 \). If \( y = 0 \), then

\[
0 = (x - 3)(x + 5)
\]

\[
x - 3 = 0 \quad x + 5 = 0
\]

\[
x = 3 \quad x = -5.
\]

**Symmetry.** If \( x \) is replaced by \(-x\), we get \( y = (-x - 3)(-x + 5) \), which does not reduce to the given equation. So there is no symmetry with respect to the \( y \)-axis. Similarly, there is no other type of symmetry.

**Asymptotes.** Since the equation is not in the form of a quotient with a variable in the denominator, there are no asymptotes.

**Extent.** \( y \) is defined for all \( x \).

**Graph.**

![Graph of y = (x-3)(x+5)](image)

(0, -15)

12. \( y = -24; x = -6, 4; \) none; none; all \( x \)

![Graph of y = -24](image)

13. **Intercepts.** If \( x = 0 \), then \( y = 0 \). If \( y = 0 \), then

\[
0 = x(x + 3)(x - 2)
\]

\[
x = 0, -3, 2.
\]

**Symmetry.** If \( x \) is replaced by \(-x\), we get \( y = -x(-x + 3)(-x - 2) \), which does not reduce to the given equation. So the graph is not symmetric with respect to the \( y \)-axis. There is no other type of symmetry.

**Asymptotes.** Since the equation is not in the form of a quotient with a variable in the denominator, there are no asymptotes.

**Extent.** \( y \) is defined for all \( x \).
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14. \( y = 0 \): \( x = 0, 1, 4 \); none; none; all \( x \)

15. Intercepts. If \( x = 0 \), \( y = 0 \); if \( y = 0 \), then

\[
x(x - 1)(x - 2)^2 = 0
\]

\[
x = 0, 1, 2.
\]

Symmetry. If \( x \) is replaced by \(-x\), we get \( y = -x(-x - 1)(-x - 2)^2 \), which does not reduce to the given equation. So there is no symmetry with respect to the \( y \)-axis. Similarly, there is no other type of symmetry.

Asymptotes. None (the equation does not have the form of a fraction).

Extent. \( y \) is defined for all \( x \).

16. \( y = 0 \): \( x = -2, 0, 3 \); none; none; all \( x \)
17. **Intercepts.** If \( x = 0 \), then \( y = 0 \). If \( y = 0 \), then

\[
0 = x(x - 1)^2(x - 2)
\]

\( x = 0, 1, 2 \).

**Symmetry.** If \( x \) is replaced with \(-x\), we get \( y = -x(-x - 1)^2(-x - 2) \), which does not reduce to the given equation. Therefore there is no symmetry with respect to the \( y \)-axis. There is no other type of symmetry.

**Asymptotes.** None (the equation does not have the form of a fraction).

**Extent.** \( y \) is defined for all \( x \).

**Graph.**

\[
\begin{array}{c}
\text{Graph} \\
\end{array}
\]

18. **Intercepts.** If \( x = 0 \), \( y = -2, 0, 3 \); none; none; all \( x \)

\[
\begin{array}{c}
\text{Graph} \\
\end{array}
\]

19. **Intercepts.** If \( x = 0 \), \( y = 1 \); if \( y = 0 \), we have

\[
0 = \frac{2}{x + 2}.
\]

This equation has no solution.

**Symmetry.** Replacing \( x \) by \(-x\), we get

\[
y = \frac{2}{-x + 2}
\]

which does not reduce to the given equation. So there is no symmetry with respect to the \( y \)-axis. Similarly, there is no other type of symmetry.

**Asymptotes.** Setting the denominator equal to 0, we get

\[
x + 2 = 0 \text{ or } x = -2.
\]

It follows that \( x = -2 \) is a vertical asymptote. Also, as \( x \) gets large, \( y \) approaches 0. So the \( x \)-axis is a horizontal asymptote.

**Extent.** To avoid division by 0, \( x \) cannot be equal to \(-2 \). So the extent is all \( x \) except \( x = -2 \).
20. $y = -1$; none; $x = 3$; $y = 0$; $x \neq 3$

21. **Intercepts.** If $x = 0$, then $y = 2$. If $y = 0$, then

$$\frac{2}{(x-1)^2} = 0.$$  

This equation has no solution.

**Symmetry.** Replacing $x$ by $-x$, we get

$$y = \frac{2}{(-x-1)^2}$$

which does not reduce to the given equation. There are no other types of symmetry.

**Asymptotes.** Setting the denominator equal to 0 gives

$$(x-1)^2 = 0 \text{ or } x = 1.$$  

It follows that $x = 1$ is a vertical asymptote. Also, as $x$ gets large, $y$ approaches 0. So the $x$-axis is a horizontal asymptote.

**Extent.** To avoid division by 0, $x$ cannot be equal to 1. So the extent is the set of all $x$ except $x = 1$. 

Graph.
22. \( y = -\frac{1}{4}; \) none; none; \( x = -2; y = 0; x \neq -2 \)

\[
\begin{array}{c}
\text{y} \\
\hline
-6 & -2 & 0 & 2 \\
\hline
\text{x}
\end{array}
\]

23. **Intercepts.** If \( x = 0 \), then \( y = 0 \). If \( y = 0 \), then

\[
0 = \frac{x^2}{x - 1}.
\]

The only solution is \( x = 0 \).

**Symmetry.** Replacing \( x \) by \(-x\) yields

\[
y = \frac{(-x)^2}{-x - 1} = \frac{x^2}{-x - 1}
\]

which is not the same as the given equation. So the graph is not symmetric with respect to the \( y \)-axis. Replacing \( y \) by \(-y\), we have

\[
-y = \frac{x^2}{x - 1}
\]

which does not reduce to the given equation. So the graph is not symmetric with respect to the \( x \)-axis.

Similarly, there is no symmetry with respect to the origin.

**Asymptotes.** Setting the denominator equal to 0, we get \( x - 1 = 0 \), or \( x = 1 \). So \( x = 1 \) is a vertical asymptote. There are no horizontal asymptotes.

(Observation: for very large \( x \) the 1 in the denominator becomes insignificant. So the graph gets ever closer to \( y = \frac{x^2}{x} = x \); the line \( y = x \) is a slant asymptote.)

**Extent.** To avoid division by 0, \( x \) cannot be equal to 1. So the extent is all \( x \) except \( x = 1 \).

**Graph.**

24. \( y = 0; x = 0; \) none; \( x = -2; y = 1; x \neq -2 \)

\[
\begin{array}{c}
\text{y} \\
\hline
-5 & -3 & 1 & 2 \\
\hline
\text{x}
\end{array}
\]
25. **Intercepts.** If \( x = 0 \), then \( y = -1/2 \). If \( y = 0 \), then

\[
0 = \frac{x + 1}{(x - 1)(x + 2)}.
\]

The only solution is \( x = -1 \).

**Symmetry.** Replacing \( x \) by \(-x\) yields

\[
y = \frac{-x + 1}{(-x - 1)(-x + 2)}
\]

which is not the same as the given equation. There are no types of symmetry.

**Asymptotes.** Setting the denominator equal to 0, we get \((x - 1)(x + 2) = 0\). So \( x = 1 \) and \( x = -2 \) are the vertical asymptotes. As \( x \) gets large, \( y \) approaches 0, so the \( x \)-axis is a horizontal asymptote.

**Extent.** To avoid division by 0, the extent is all \( x \) except \( x = 1 \) and \( x = -2 \).

**Graph.**

![Graph](image)

26. \( y = 0, x = 1, 0; \) none; \( x = -1, 2, y = 1; x \neq -1, 2 \)

![Graph](image)

27. **Intercepts.** If \( x = 0 \), then \( y = 4 \). If \( y = 0 \), then

\[
\frac{x^2 - 4}{x^2 - 1} = 0
\]

\[
x^2 - 4 = 0 \quad \text{multiplying by } x^2 - 1
\]

\[
x = \pm 2. \quad \text{solution}
\]

**Symmetry.** Replacing \( x \) by \(-x\) reduces to the given equation. So there is symmetry with respect to the \( y \)-axis. There is no other type of symmetry.

**Asymptotes.** Vertical: setting the denominator equal to 0, we have

\[
x^2 - 1 = 0 \text{ or } x = \pm 1.
\]

**Horizontal:** dividing numerator and denominator by \( x^2 \), the equation becomes

\[
y = \frac{1 - \frac{4}{x}}{1 - \frac{1}{x}}
\]
1.4. CURVE SKETCHING

As $x$ gets large, $y$ approaches 1. So $y = 1$ is a horizontal asymptote.

Extent. All $x$ except $x = \pm 1$ (to avoid division by 0).

Graph.

28. $y = \frac{1}{x}, x = \pm 1; y$-axis; $x = \pm 2, y = 1; x \neq \pm 2$

29. Intercepts. If $x = 0$, then $y^2 = \frac{-4}{1} = 4$, or $y = \pm 2$. If $y = 0$, then

$$0 = \frac{x^2 - 4}{x^2 - 1}$$

which is possible only if $x^2 - 4 = 0$, or $x = \pm 2$.

Symmetry. The even powers on $x$ and $y$ tell us that if $x$ is replaced by $-x$ and $y$ is replaced by $-y$, the resulting equation will reduce to the given equation. The graph is therefore symmetric with respect to both axes and the origin.

Asymptotes. Vertical: setting the denominator equal to 0, we get

$$x^2 - 1 = 0 \text{ or } x = \pm 1.$$ 

Horizontal: dividing numerator and denominator by $x^2$, we get

$$y^2 = \frac{1 - \frac{4}{x^2}}{1 - \frac{1}{x^2}}.$$ 

The right side approaches 1 as $x$ gets large. Thus $y^2$ approaches 1, so that $y = \pm 1$ are the horizontal asymptotes.
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Extended. From

\[ y = \pm \sqrt{\frac{x^2 - 4}{x^2 - 1}} \]

we conclude that

\[ \frac{x^2 - 4}{x^2 - 1} = \frac{(x - 2)(x + 2)}{(x - 1)(x + 1)} \geq 0. \]

Since the signs change only at \( x = 2, -2, 1, \) and \(-1\), we need to use arbitrary "test values" between these points. The results are summarized in the following chart.

<table>
<thead>
<tr>
<th>test values</th>
<th>( x - 2 )</th>
<th>( x - 1 )</th>
<th>( x + 1 )</th>
<th>( x + 2 )</th>
<th>( \frac{(x - 2)(x + 2)}{(x - 1)(x + 1)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &gt; 2 )</td>
<td>3</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( 1 &lt; x &lt; 2 )</td>
<td>( 3/2 )</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( -1 &lt; x &lt; 1 )</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( -2 &lt; x &lt; -1 )</td>
<td>( -3/2 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( x &lt; -2 )</td>
<td>-3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Note that the fraction is positive only when \( x > 2, -1 < x < 1 \) and \( x < -2 \). Since \( y = 0 \) when \( x = \pm 2 \), the extent is \( x \geq 2, -1 < x < 1, x \leq -2 \).

Graph.

\[ y = \pm \frac{1}{x} \quad x = \pm 1; \text{ both axes; } x = \pm 2, y = \pm 1; x < -2, -1 \leq x \leq 1, x > 2 \]

30. \( y = \pm \frac{1}{x} \quad x = \pm 1; \text{ both axes; } x = \pm 2, y = \pm 1; x < -2, -1 \leq x \leq 1, x > 2 \)

31. Intercepts. If \( x = 0 \), \( y^2 = (-3)(5) = -15 \), or \( y = \pm \sqrt{15} \), which is a pure imaginary number. If \( y = 0 \), \( \frac{(x - 3)(x + 5)}{x} = 0 \)

\[ x = 3, -5. \]

Symmetry. Replacing \( y \) by \( -y \), we get \((-y)^2 = (x - 3)(x + 5)\), which reduces to the given equation. Hence the graph is symmetric with respect to the \( x \)-axis.

Asymptotes. None (no fractions).

Extent. From \( y = \pm \sqrt{(x - 3)(x + 5)} \), we conclude that \((x - 3)(x + 5) \geq 0 \). If \( x \geq 3 \), \( (x - 3)(x + 5) \geq 0 \). If \( x \leq -5 \), \( (x - 3)(x + 5) \geq 0 \), since both factors are negative (or zero).

If \(-5 < x < 3 \), \((x - 3)(x + 5) < 0 \). [For example, if \( x = 0 \), we get \((-3)(5) = -15 \).] These observations are summarized in the following chart.
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1.4. CURVE SKETCHING

<table>
<thead>
<tr>
<th>test values</th>
<th>$x - 3$</th>
<th>$x + 5$</th>
<th>$(x - 3)(x + 5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &gt; 3$</td>
<td>4</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$-5 &lt; x &lt; 3$</td>
<td>0</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$x &lt; -5$</td>
<td>-6</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Extent: $x \leq -5$, $x \geq 3$

Graph.

32. $y = 0$, $x = 0$; $x$-axis; $x = -2$, $y = \pm 1$; $x < -2$, $x \geq 0$

33. Intercepts. If $x = 0$, $y = 0$; if $y = 0$, $x = 0$.

Symmetry. Replacing $y$ by $-y$ leaves the equation unchanged. So there is symmetry with respect to the $x$-axis. There is no other type of symmetry.

Asymptotes. Vertical: setting the denominator equal to 0, we get

$$(x - 3)(x - 2) = 0$$

or $x = 3, 2$.

Horizontal: as $x$ gets large, $y$ approaches 0 ($x$-axis).

Extent. From

$$y = \pm \sqrt{\frac{x}{(x - 3)(x - 2)}}$$

we conclude that

$$\frac{x}{(x - 3)(x - 3)} \geq 0.$$ 

Since signs change only at $x = 0, 2$ and $3$, we need to use “test values” between these points.

The results are summarized in the following chart.

<table>
<thead>
<tr>
<th>test values</th>
<th>$x$</th>
<th>$x - 2$</th>
<th>$x - 3$</th>
<th>$\frac{x}{(x - 3)(x - 2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 0$</td>
<td>-1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$0 &lt; x &lt; 2$</td>
<td>1</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$2 &lt; x &lt; 3$</td>
<td>5/2</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$x &gt; 3$</td>
<td>4</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

So the inequality is satisfied for $0 < x < 2$ and $x > 3$. In addition, $y = 0$ when $x = 0$. 

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So the extent is $0 \leq x < 2$ and $x > 3$.

Graph.

34. $x = -1$; $x$-axis; $x = -2$, $1$, $y = 0$; $-2 < x \leq -1$, $x > 1$

35. $C = \frac{10^{-2}C_1}{C_1 + 10^{-2}}$, $C_1 \geq 0$

The only intercept is the origin. Dividing numerator and denominator by $C_1$, the equation becomes

$$C = \frac{10^{-2}}{1 + 10^{-2}/C_1}.$$  

As $C_1$ gets large, $C$ approaches $10^{-2}$; so $C = 10^{-2}$ is a horizontal asymptote.

See graph in answer section of book.

37. Intercepts. If $t = 0$, $S = 0$; if $S = 0$, we get

$$0 = 60t - 5t^2$$

or $t = 0, 12$.

Symmetry. None.

Asymptotes. None.

Extent. $t \geq 0$ by assumption.

Graph. See graph in answer section of book.

38.

39. Extent $L \geq 0$.

See graph in answer section of book.
1.5 Curves with Graphing Utilities

Graphs are from the answer section of the book.

1. If \( y = 0 \), then

\[ x^2(x - 1)(x - 2) = 0. \]

Setting each factor equal to 0, we get

\[ x = 0, 1, 2. \]
4. $-2, -1, 0, 1$

\[ [-3, 3] \text{ by } [-0.5, 0.5] \]

5. \[ x^4 - 2x^3 = 0 \]
   \[ x^3(x - 2) = 0 \]
   \[ x = 0, 2 \]

\[ [-1, 3] \text{ by } [-2, 2] \]

6. \[ \pm \frac{\sqrt{6}}{2} \]

\[ [-2, 2] \text{ by } [-15, 15] \]

8. \[ 0, \frac{1}{2} \]

\[ [-1, 1] \text{ by } [-0.2, 0.2] \]
9. Domain: \( x \geq 0 \) (to avoid imaginary values).
   Vertical asymptotes: None. (The denominator is always positive, that is, \( 1 + \sqrt{x} \neq 0 \).)

10. \( x = -1, x > -1 \)

12. \( x = 1, x \geq 0 \)

13. To find the vertical asymptotes, we set the denominator equal to 0:
    \[
    2x^2 - 3 = 0 \\
    2x^2 = 3 \\
    x^2 = \frac{3}{2} \\
    x = \pm \sqrt{\frac{3}{2}} \\
    \]
    Domain: \( y \) is defined for all \( x \) except \( x = \pm \sqrt{\frac{3}{2}} \).
14. \[ x = \pm \frac{\sqrt{10}}{2}, x \neq \pm \frac{\sqrt{10}}{2} \]

16. \[ x = 0, x \geq -1 \]

17. See graph in answer section of book.

18. \((-0.65, 4.36)\)
1.7 The Circle

1. Since \((h, k) = (0, 0)\) and \(r = 4\), we get from the form

\[(x - h)^2 + (y - k)^2 = r^2\]

the equation

\[x^2 + y^2 = 16.\]

2. Since \((h, k) = (0, 0)\) and \(r = 8\), we get from the form \((x - h)^2 + (y - k)^2 = r^2\) the equation \(x^2 + y^2 = 64\).

3. The radius of the circle is the distance from the origin to \((-6, 8)\). Hence

\[r^2 = (0 + 6)^2 + (0 - 8)^2 = 100.\]

From the standard form of the equation of the circle we get

\[(x - 0)^2 + (y - 0)^2 = 100 \quad \text{center: } (0, 0)\]

\[x^2 + y^2 = 100.\]

4. The radius of the circle is the distance from the origin to the point \((1, -4)\). Hence

\[r^2 = (0 - 1)^2 + (0 + 4)^2 = 17\]

Equation: \(x^2 + y^2 = 17\), \((h, k) = (0, 0)\)

5. \[(x - h)^2 + (y - k)^2 = r^2\]

\[(x + 2)^2 + (y - 5)^2 = 1^2\]

\[x^2 + y^2 + 4x - 10y + 28 = 0\]
6. \((x-h)^2 + (y-k)^2 = r^2\)
   \((x-2)^2 + (y+3)^2 = (\sqrt{2})^2\)
   \(x^2 - 4x + 4 + y^2 + 6y + 9 = 2\)
   \(x^2 + y^2 - 4x + 6y + 11 = 0\)

7. The radius is the distance from \((-1, -4)\) to the origin:

   \[r^2 = (-1 - 0)^2 + (-4 - 0)^2 = 1 + 16 = 17.\]

   Hence,
   \[(x+1)^2 + (y+4)^2 = 17\]
   \[(x-h)^2 + (y-k)^2 = r^2\]
   \[x^2 + 2x + 1 + y^2 + 8y + 16 = 17\]
   \[x^2 + y^2 + 2x + 8y = 0.\]

8. The radius is the distance from the center to a point on the circle. Thus

   \[r^2 = (5 - 3)^2 + (10 - 4)^2 = 40\]
   \[(x - 3)^2 + (y - 4)^2 = 40\]
   \[x^2 - 6x + 9 + y^2 - 8y + 16 = 40\]
   \[x^2 + y^2 - 6x - 8y - 15 = 0\]

9. Diameter: distance from \((-2, -6)\) to \((1, 5)\). Hence

   \[r = \frac{1}{2}\sqrt{(-2 - 1)^2 + (-6 - 5)^2} = \frac{1}{2}\sqrt{9 + 121} = \frac{1}{2}\sqrt{130}\]

   and thus,

   \[r^2 = \frac{1}{4}(130) = \frac{65}{2}.\]

   Center: midpoint of the line segment, whose coordinates are

   \[\left(-\frac{2 + 1}{2}, -\frac{6 + 5}{2}\right) = \left(-\frac{1}{2}, -\frac{11}{2}\right).\]

   Thus

   \[(x + \frac{1}{2})^2 + (y + \frac{1}{2})^2 = \frac{65}{2}\]
   \[x^2 + x + \frac{1}{4} + y^2 + y + \frac{1}{4} = \frac{65}{2}\]
   \[x^2 + y^2 + x + y - 32 = 0.\]

10. Distance from \((-1, 2)\) to the y-axis: 1

    \[(x + 1)^2 + (y - 2)^2 = 1\]

11. Since \(r = 5\), we get

    \[(x - 4)^2 + (y + 5)^2 = 25\] or \[x^2 + y^2 - 8x + 10y + 16 = 0\]
12. Distance to the line \( y = 1 \): 
\[
(x - 3)^2 + (y - 4)^2 = 9
\]

13. From the diagram, \( r^2 = 1^2 + 1^2 = 2 \); so \( x^2 + y^2 = 2 \).

14. Since the circle is tangent to the \( y \)-axis with radius 2, \( h = 2 \) or \( h = -2 \). The equation \( y = \frac{3}{2}x \) yields two possibilities for the center: \( (2, 3) \) and \( (-2, -3) \). Thus

\[
(x - 2)^2 + (y - 3)^2 = 2^2 \quad \text{and} \quad (x + 2)^2 + (y + 3)^2 = 2^2
\]

or

\[
x^2 + y^2 - 4x - 6y + 9 = 0 \quad \text{and} \quad x^2 + y^2 + 4x + 6y + 9 = 0
\]

15. \( x^2 + y^2 - 2x - 2y - 2 = 0 \)

\[
x^2 - 2x + y^2 - 2y = 2
\]

We now add to each side the square of one-half the coefficient of \( x \):

\[
\left[ \frac{1}{2}(-2) \right]^2 = 1
\]

\[
x^2 - 2x + 1 + y^2 - 2y = 2 + 1.
\]

Similarly, we add 1 (the square of one-half the coefficient of \( y \)):

\[
(x - 1)^2 + (y - 1)^2 = 4.
\]

Center: \((h, k) = (1, 1)\); radius: \(\sqrt{4} = 2\).

16. \( x^2 + y^2 - 2x - 4y + 4 = 0 \)

\[
x^2 - 2x + y^2 - 4y = -4
\]

We now add to each side the square of one-half the coefficient of \( x \): \( \left[ \frac{1}{2}(-2) \right]^2 = 1 \):

\[
x^2 - 2x + 1 + y^2 - 4y = -4 + 1
\]

Similarly, we add to each side the square of one-half the coefficient of \( y \): \( \left[ \frac{1}{2}(-4) \right]^2 = 4 \)

\[
x^2 - 2x + 1 + y^2 - 4y + 4 = -4 + 1 + 4
\]

\((x - 1)^2 + (y - 2)^2 = 1\); Center: \((h, k) = (1, 2)\); radius: \(\sqrt{1} = 1\).

17. \( x^2 + y^2 + 4x - 8y + 4 = 0 \)

\[
x^2 + 4x + y^2 - 8y = -4
\]

Since

\[
\left( \frac{1}{2} \cdot 4 \right)^2 = 4 \quad \text{and} \quad \left[ \frac{1}{2}(-8) \right]^2 = 16
\]

we get

\[
(x^2 + 4x + 4) + (y^2 - 8y + 16) = -4 + 4 + 16
\]

\[
(x + 2)^2 + (y - 4)^2 = 16.
\]

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The equation can be written

\[ (x - (-2))^2 + (y - 4)^2 = 4^2. \]

It follows that

\((h, k) = (-2, 4)\) and \(r = 4\).

18. \(x^2 + y^2 + 2x + 6y + 3 = 0\)

\(x^2 + 2x + y^2 + 6y = -3\)

Observe that \(\left[\frac{1}{2}(2)\right]^2 = 1\) and \(\left[\frac{1}{2}(6)\right]^2 = 9\). Adding 1 and 9 to each side, we get

\[ x^2 + 2x + 1 + y^2 + 6y + 9 = -3 + 1 + 9 \]

or

\[ (x + 1)^2 + (y + 3)^2 = 7 \]

Center: \((-1, -3)\); radius: \(\sqrt{7}\).

19. \(x^2 + y^2 - 4x + y + \frac{9}{4} = 0\)

\(x^2 - 4x + y^2 + y = -\frac{9}{4}\)

We add to each side the square of one-half the coefficient of \(x\):

\[ \left[\frac{1}{2}(-4)\right]^2 = 4 \]. This gives

\[ x^2 - 4x + 4 + y^2 + y = -\frac{9}{4} + 4 \]

Similarly, we add the square of one-half the coefficient of \(y\):

\[ \left[\frac{1}{2}(1)\right]^2 = \frac{1}{4} \]. This gives

\[ x^2 - 4x + 4 + y^2 + y + \frac{1}{4} = -\frac{9}{4} + 4 + \frac{1}{4} \]

\[ (x - 2)^2 + (y + \frac{1}{2})^2 = -\frac{8}{4} + 4 \]

\[ (x - 2)^2 + (y + \frac{1}{2})^2 = 2 \]

Center: \((2, -\frac{1}{2})\); radius: \(\sqrt{2}\).

20. \(x^2 + y^2 + x - 4y + \frac{5}{4} = 0\)

\(x^2 + x + y^2 - 4y = -\frac{5}{4}\)

Add to each side: \(\left[\frac{1}{2}(1)\right]^2 = \frac{1}{4}\) and \(\left[\frac{1}{2}(-4)\right]^2 = 4\)

\[ x^2 + x + \frac{1}{4} + y^2 - 4y + 4 = -\frac{5}{4} + \frac{1}{4} + 4 \]

\[ (x + \frac{1}{2})^2 + (y - 2)^2 = 3 \]; Center: \((-\frac{1}{2}, 2)\); radius: \(\sqrt{3}\).

21. \(4x^2 + 4y^2 - 8x - 12y + 9 = 0\)

\(x^2 + y^2 - 2x - 3y + \frac{9}{4} = 0\) dividing by 4

\(x^2 - 2x + y^2 - 3y = -\frac{9}{4}\)

Add to each side: \(\left[\frac{1}{2}(-2)\right]^2 = 1\) and \(\left[\frac{1}{2}(-3)\right]^2 = \frac{9}{4}\) to get

\[ (x - 1)^2 + (y - \frac{3}{2})^2 = 1 \]

Center: \((1, \frac{3}{2})\); radius: 1.
22. \(4x^2 + 4y^2 - 20x - 8y + 25 = 0\)

First divide both sides by 4:
\[
x^2 + y^2 - 5x - 2y + \frac{25}{4} = 0
\]
\[
x^2 - 5x + y^2 - 2y = -\frac{25}{4}
\]
Add to each side: \(\left[\frac{1}{2}(-5)\right]^2 = \frac{25}{4}\) and \(\left[\frac{1}{2}(-2)\right]^2 = 1\)

\[
x^2 - 5x + \frac{25}{4} + y^2 - 2y + 1 = -\frac{25}{4} + \frac{25}{4} + 1
\]

\[
(x - \frac{5}{2})^2 + (y - 1)^2 = 1; \text{ Center: } \left(\frac{5}{2}, 1\right); \text{ radius: 1}
\]

23. \(x^2 + y^2 + 4x - 2y - 4 = 0\)

\[
x^2 + 4x + y^2 - 2y = 4
\]

Note that \(\left(\frac{1}{2} \cdot 4\right)^2 = 4\) and \(\left[\frac{1}{2}(-2)\right]^2 = 1\).

Adding 4 and 1, respectively, we get
\[
(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 4 + 1
\]
\[
(x + 2)^2 + (y - 1)^2 = 9.
\]

The equation can be written
\[
[x - (-2)]^2 + (y - 1)^2 = 3^2.
\]

So the center is \((-2, 1)\) and the radius is 3.

24. \(x^2 + y^2 + 2x + 8y + 1 = 0\)

\[
x^2 + 2x + y^2 + 8y = -1
\]

Add to each side: \(\left[\frac{1}{2}(2)\right]^2 = 1\) and \(\left[\frac{1}{2}(8)\right]^2 = 16\)

\[
x^2 + 2x + 1 + y^2 + 8y + 16 = -1 + 1 + 16
\]

\[
(x + 1)^2 + (y + 4)^2 = 16; \text{ Center: } (-1, -4); \text{ radius: 4}
\]

25. \(x^2 + y^2 - x - 2y + \frac{1}{4} = 0\)

\[
x^2 - x + y^2 - 2y = -\frac{1}{4}
\]

Add to each side: \(\left[\frac{1}{2}(-1)\right]^2 = \frac{1}{4}\) and \(\left[\frac{1}{2}(-2)\right]^2 = 1\) to get

\[
(x^2 - x + \frac{1}{4}) + (y^2 - 2y + 1) = -\frac{1}{4} + \frac{1}{4} + 1
\]
\[
(x - \frac{1}{2})^2 + (y - 1)^2 = 1.
\]

Center: \(\left(\frac{1}{2}, 1\right)\); radius: 1.

26. \(x^2 + y^2 - 6x + 8y + 19 = 0\)

\[
x^2 - 6x + y^2 - 8y = -19
\]

Add to each side: \(\left[\frac{1}{2}(-6)\right]^2 = 9\) and \(\left[\frac{1}{2}(-8)\right]^2 = 16\)

\[
x^2 - 6x + 9 + y^2 - 8y + 16 = -19 + 9 + 16
\]

\[
(x - 3)^2 + (y - 4)^2 = 6; \text{ Center: } (3, 4); \text{ radius: } \sqrt{6}
\]
27. \[ x^2 + y^2 - 4x + y + \frac{9}{4} = 0 \]
\[ x^2 - 4x + y^2 + y = -\frac{9}{4} \]

Note that \[ \left(\frac{1}{2}(-4)\right)^2 = 4 \] and \[ \left(\frac{1}{2} \cdot 1\right)^2 = \frac{1}{4} \].

Adding 4 and \( \frac{1}{4} \), respectively, we get
\[ (x - 2)^2 + (y + \frac{1}{2})^2 = 2. \]

The equation can be written
\[ (x - 2)^2 + \left[ y - \left(-\frac{1}{2}\right)\right]^2 = (\sqrt{2})^2. \]

Center: \((2, -\frac{1}{2})\); radius: \(\sqrt{2}\).

28. \[ x^2 + y^2 + x - y - \frac{1}{2} = 0 \]
\[ x^2 + x + y^2 - y = \frac{1}{2} \]

Add to each side: \[ \left(\frac{1}{2}(1)\right)^2 = \frac{1}{4} \] and \[ \left(\frac{1}{2}(-1)\right)^2 = \frac{1}{4} \]
\[ x^2 + x + \frac{1}{4} + y^2 - y + \frac{1}{4} = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \]
\[ (x + \frac{1}{2})^2 + (y - \frac{1}{2})^2 = 1; \text{ Center: } \left(-\frac{1}{2}, \frac{1}{2}\right); \text{ radius: } 1 \]

29. \[ 4x^2 + 4y^2 + 12x + 16y + 5 = 0 \]
\[ x^2 + y^2 + 3x + 4y + \frac{5}{4} = 0 \]
\[ x^2 + 3x + y^2 + 4y = -\frac{5}{4} \]

Add to each side: \[ \left(\frac{1}{2} \cdot 3\right)^2 = \frac{9}{4} \] and \[ \left(\frac{1}{2} \cdot 4\right)^2 = 4 \] to get
\[ x^2 + 3x + \frac{9}{4} + y^2 + 4y + 4 = -\frac{5}{4} + \frac{9}{4} + 4 \]
\[ (x + \frac{3}{2})^2 + (y + 2)^2 = 5. \]

Center: \((-\frac{3}{2}, -2)\); radius: \(\sqrt{5}\).

30. \[ 36x^2 + 36y^2 - 144x - 120y + 219 = 0 \]

First divide both sides by 36:
\[ x^2 + y^2 - 4x - \frac{10}{3}y + \frac{219}{36} = 0 \]
\[ x^2 - 4x + y^2 - \frac{10}{3}y = -\frac{219}{36} \]

Add to each side: \[ \left(\frac{1}{2}(-4)\right)^2 = 4 \] and \[ \left(\frac{1}{2}\left(-\frac{10}{3}\right)\right)^2 = \frac{25}{9} \]
\[ x^2 - 4x + 4 + y^2 - \frac{10}{3}y + \frac{25}{9} = -\frac{219}{36} + 4 + \frac{25}{9} = \frac{-219 + 144 + 100}{36} \]
\[ (x - 2)^2 + \left( y - \frac{5}{3} \right)^2 = \frac{25}{36}; \text{ Center: } \left(2, \frac{5}{3}\right); \text{ radius: } \frac{5}{6} \]

31. \[ 4x^2 + 4y^2 - 20x - 4y + 26 = 0 \]
\[ x^2 + y^2 - 5x - y + \frac{26}{4} = 0 \]

dividing by 4
\[ x^2 - 5x + y^2 - y = -\frac{26}{4} \]
\[ x^2 - 5x + \frac{25}{4} + y^2 - y + \frac{1}{4} = -\frac{26}{4} + \frac{25}{4} + \frac{1}{4} \]
\[ (x - \frac{5}{2})^2 + (y - \frac{1}{2})^2 = 0 \]

Locus is the single point \(\left(\frac{5}{2}, \frac{1}{2}\right)\).
32. \[x^2 + y^2 + 4x - 2y + 7 = 0\]
   \[x^2 + 4x + y^2 - 2y = -7\]
   \[x^2 + 4x + 4 + y^2 - 2y + 1 = -7 + 4 + 1\]
   \[(x + 2)^2 + (y - 1)^2 = -2\]

Imaginary circle

33. \[x^2 + y^2 - 6x + 8y + 25 = 0\]
   \[x^2 - 6x + y^2 + 8y = -25\]
   \[(x^2 - 6x + 9) + (y^2 + 8y + 16) = -25 + 9 + 16\]
   \[(x - 3)^2 + (y + 4)^2 = 0\]

Locus is the single point \((3, -4)\).

34. \[x^2 + y^2 + 2x + 4y + 5 = 0\]
   \[x^2 + 2x + y^2 + 4y = -5\]
   \[x^2 + 2x + 1 + y^2 + 4y + 4 = -5 + 1 + 4\]
   \[(x + 1)^2 + (y + 2)^2 = 0\]

Point circle: \((-1, -2)\)

35. \[x^2 + y^2 - 6x - 8y + 30 = 0\]
   \[x^2 - 6x + y^2 - 8y = -30\]
   \[x^2 - 6x + 9 + y^2 - 8y + 16 = -30 + 9 + 16\]
   \[(x - 3)^2 + (y - 4)^2 = -5\]

(imaginary circle)

36. \[x^2 + y^2 - 6x + 4y + 13 = 0\]
   \[x^2 - 6x + y^2 + 4y = -13\]
   \[x^2 - 6x + 9 + y^2 + 4y + 4 = -13 + 9 + 4\]
   \[(x - 3)^2 + (y + 2)^2 = 0\]

Point circle: \((3, -2)\)

37. \[x^2 + y^2 - x + 4y + \frac{17}{4} = 0\]
   \[x^2 - x + y^2 + 4y = -\frac{17}{4}\]

We add to each side \(\left[\frac{1}{2}(-1)\right]^2\) and \(\left[\frac{1}{2}(4)\right]^2\):
\[x^2 - x + \frac{1}{4} + y^2 + 4y + 4 = -\frac{17}{4} + \frac{1}{4} + 4\]
\[(x - \frac{1}{2})^2 + (y + 2)^2 = 0\]

(point circle)

38. From the given circle, we have
   \[x^2 - 6x + y^2 - 4y = 12\]
   \[x^2 - 6x + 9 + y^2 - 4x + 4 = 12 + 9 + 4\]
   \[(x - 3)^2 + (y - 2)^2 = 25\]

Center: \((3, 2)\); Desired circle: \((x - 3)^2 + (y - 2)^2 = 9\)

39. \[x^2 + y^2 = (2.00)^2 = 4.00; \ x^2 + y^2 = (3.40)^2 = 11.6\]

40. Center: \((5.00, 5.00)\); radius: 2.10
   \[(x - 5.00)^2 + (y - 5.00)^2 = (2.10)^2\]
   \[x^2 - 10.0x + 25.00 + y^2 - 10.0y + 25.00 = 4.41\]
   \[x^2 + y^2 - 10.0x - 10.0y + 45.6 = 0\]

41. The radius is 22,300 + 4000 = 26,300 mi.
43. \((h,k) = (0,0)\) and \(r = \frac{3}{2}\) ft: \(x^2 + y^2 = \frac{9}{4}\) and \(y = \sqrt{\frac{9}{4} - x^2}\)

### 1.8 The Parabola

1. Since the focus is on the \(x\)-axis, the form is \(y^2 = 4px\). Since the focus is at \((3,0)\), \(p = 3\) (positive). Thus \(y^2 = 4(3)x\), or \(y^2 = 12x\).

2. Since the focus is on the \(x\)-axis, the form is \(y^2 = 4px\). Since the focus is at \((-3,0)\), \(p = -3\) (negative). Thus \(y^2 = 4(-3)x\), or \(y^2 = -12x\).

3. Since the focus is on the \(y\)-axis, the form is \(x^2 = 4py\). Since the focus is at \((0,-5)\), \(p = -5\) (negative). Thus \(x^2 = 4(-5)y\), or \(x^2 = -20y\).

4. Since the focus is on the \(y\)-axis, the form is \(x^2 = 4py\). Since the focus is at \((0,4)\), \(p = 4\) (positive). Thus \(x^2 = 4(4)y\), or \(x^2 = 16y\).

5. Since the focus is on the \(x\)-axis, the form is \(y^2 = 4px\). The focus is on the left side of the origin, at \((-4,0)\). So \(p = -4\) (negative). It follows that \(y^2 = 4(-4)x\), or \(y^2 = -16x\).

6. Since the focus is on the \(y\)-axis, the form is \(x^2 = 4py\). Since the focus is at \((0,-6)\), \(p = -6\) (negative). Thus \(x^2 = 4(-6)y\), or \(x^2 = -24y\).

7. Since the directrix is \(x = -1\), the focus is at \((1,0)\). So the form is \(y^2 = 4px\) with \(p = 1\), and the equation is \(y^2 = 4x\).

8. Since the directrix is \(y = 2\), the focus is at \((0,-2)\), so that \(p = -2\). From the form \(x^2 = 4py\), we get \(x^2 = -8y\).

9. Since the directrix is \(x = 2\), the focus is at \((-2,0)\). So the form is \(y^2 = 4px\) with \(p = -2\). Thus \(y^2 = -8x\).

10. Directrix: \(x = -2\); focus: \((2,0)\); form: \(y^2 = 4px\); equation: \(y^2 = 8x\)

11. Form: \(y^2 = 4px\). Substituting the coordinates of the point \((-2, -4)\), we get

\[
(-4)^2 = 4p(-2) \quad \text{and} \quad 4p = -8.
\]

Thus \(y^2 = -8x\).

12. Form: \(x^2 = 4py\); the coordinates of the point \((-1, 1)\) satisfy the equation: \((-1)^2 = 4p(1)\), so \(4p = 1\), and we get \(x^2 = y\).

13. The form is either

\[
y^2 = 4px \quad \text{or} \quad x^2 = 4py.
\]

Substituting the coordinates of the point \((1, 1)\), we get

\[
1^2 = 4p \cdot 1 \quad \text{or} \quad 1^2 = 4p \cdot 1.
\]

In either case, \(p = \frac{1}{4}\). So the equations are \(y^2 = x\) and \(x^2 = y\).
14. Both forms are possible; substituting the coordinates of the point \((2, -1)\), we get
\[
\begin{align*}
x^2 &= 4py \\
2^2 &= 4p(-1) \\
4p &= -4 \\
x^2 &= -4y
\end{align*}
\]
\[
\begin{align*}
y^2 &= 4px \\
(-1)^2 &= 4p(2) \\
4p &= \frac{1}{2} \\
y^2 &= \frac{1}{2}x
\end{align*}
\]

15. The form is either \(y^2 = 4px\) or \(x^2 = 4py\). Substituting the coordinates of the point \((-2, 4)\), we get
\[
4^2 = 4p(-2) \quad \text{or} \quad (-2)^2 = 4p(4)
\]
The respective values of \(p\) are \(-2\) and \(\frac{1}{2}\); so the equations are \(y^2 = -8x\) and \(x^2 = y\).

16. Form: \(y^2 = 4px\) or \(x^2 = 4py\). Substituting \((3, -5)\):
\[
\begin{align*}
25 &= 4p(3) \quad \text{or} \quad 9 = 4p(-5) \\
p &= \frac{25}{12} \quad \text{or} \quad p = \frac{-9}{20}
\end{align*}
\]
The equations are \(y^2 = \frac{25}{3}x\) and \(x^2 = -\frac{9}{5}y\).

17. From \(x^2 = 12y\), we have \(x^2 = 4(3y)\). Thus \(p = 3\) and the focus is at \((0, 3)\).

18. \(x^2 = 20y\)
\[
\begin{align*}
x^2 &= 4(5)y \\
p &= 5 \\
\text{focus}: \ (0, 5)
\end{align*}
\]

19. From \(x^2 = -8y\), we have \(x^2 = 4(-2)y\). So \(p = -2\) and the focus is at \((0, -2)\).
20. \( x^2 = -24y \)
\( x^2 = 4(-6)y \)
\( p = -6 \)

\[ y = 6 \]
\[ x = 0 \]
\[ (0, -6) \]

21. \( y^2 = 24x = 4(6)x; p = 6 \) and the focus is at \((6, 0)\).

\[ (6, 0)^2 \]

22. \( y^2 = 12x \)
\( y^2 = 4(3)x \)
\( p = 3 \)

\[ x = -3 \]
\[ x = 0 \]
\[ (3, 0) \]

23. From \( y^2 = -4x \), \( y^2 = 4(-1)x \). So \( p = -1 \) and the focus is at \((-1, 0)\).

\[ (-1, 0) \]

24. \( y^2 = -12x \)
\( y^2 = 4(-3)x \)
\( p = -3 \)

\[ (3, 0) \]

\[ x = 3 \]
25. \( x^2 = 4y = 4(1)y; \ p = 1 \) and the focus is at \((0, 1)\).

\[
\begin{array}{c}
\text{(0,1)} \\
0 \\
x
\end{array}
\]

26. \( x^2 = -12y \)
\( x^2 = 4(-3)y \)

Focus: \((0, -3)\), Directrix: \(y - 3 = 0\)

27. From \( y^2 = 9x, \ y^2 = 4\left(\frac{3}{4}\right)x \) (inserting 4). So \( p = \frac{9}{4} \) and the focus is at \( \left(\frac{9}{4}, 0\right)\).

28. \( y^2 = 10x \)
\( y^2 = 4\left(\frac{5}{2}\right)x \)
\( p = \frac{5}{2} \)

Focus: \( \left(\frac{5}{2}, 0\right)\), Directrix: \(x + \frac{5}{2} = 0\)

29. \( y^2 = -x = 4(-\frac{1}{4})x; \ p = -\frac{1}{4} \) and the focus is at \( (-\frac{1}{4}, 0)\).

30. \( x^2 = \frac{3}{2}y \)
\( x^2 = 4\left(\frac{3}{8}\right)y \)
\( p = \frac{3}{8} \)

31. \( 3y^2 + 2x = 0 \)
\( y^2 = -\frac{2}{3}x \)
\( y^2 = 4\left(-\frac{2}{3}\cdot\frac{1}{8}\right)x \)
\( y^2 = 4\left(-\frac{1}{8}\right)x \)

So the focus is at \( (-\frac{1}{6}, 0)\).

32. \( y^2 = 2ax \)
\( y^2 = 4\left(\frac{a}{2}\right)x \)
\( p = \frac{a}{2} \)

Focus: \( \left(\frac{a}{2}, 0\right)\), Directrix: \(x + \frac{a}{2} = 0\)

33. \( x^2 = 4(3)y; \ p = 3 \). Focus: \((0, 3)\); Directrix: \(y = -3\).

\[
\begin{array}{c}
\text{(-6,3)} \\
(-6) \\
0 \\
6 \\
\end{array}
\]

\[
\begin{array}{c}
\text{(6,3)} \\
\end{array}
\]

\[
\begin{array}{c}
y = -3 \\
\end{array}
\]

Observe that the points \((6, 3)\) and \((-6, 3)\) lie on the curve because the distance to the focus must be equal to the distance to the directrix.
Circle: \((x - h)^2 + (y - k)^2 = r^2\)
\((x - 0)^2 + (y - 3)^2 = 6^2\)
\(x^2 + y^2 - 6y + 9 = 36\)
\(x^2 + y^2 - 6y - 27 = 0.\)

34. Let \((x,y)\) be a point on the parabola in Figure 1.46.

Distance to \((0,p): \sqrt{(x - 0)^2 + (y - p)^2}\)
Distance to the line \(y = -p: y - (-p) = y + p\)

By definition, the distances are equal. So
\(\sqrt{(x - 0)^2 + (y - p)^2} = y + p\)
\(x^2 + (y - p)^2 = (y + p)^2\) (squaring both sides)
\(x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2\)
\(x^2 = 4py\) after collecting terms

35. We need to find the locus of points \((x, y)\) equidistant from \((4, 1)\) and the \(y\)-axis. Since the distance from \((x, y)\) to the \(y\)-axis is \(x\) units, we get
\(\sqrt{(x - 4)^2 + (y - 1)^2} = x\)
\((x - 4)^2 + (y - 1)^2 = x^2\)
\(x^2 - 8x + 16 + y^2 - 2y + 1 = x^2\)
\(y^2 - 2y - 8x + 17 = 0.\)

36. From the figure, \(d_1 = d_2:\)
\(\sqrt{(x - 4)^2 + (y - 7)^2} = y + 1\)
\((x - 4)^2 + (y - 7)^2 = (y + 1)^2\) (squaring both sides)
\(x^2 - 8x + 16 + y^2 - 14y + 49 = y^2 + 2y + 1\)
\(x^2 - 8x - 16y + 64 = 0\)

37. If the origin is the lowest point on the cable, then the top of the right supporting tower is at \((100, 70)\).

From the equation \(x^2 = 4py\), we get
\((100)^2 = 4p(70)\)
\(4p = \frac{10,000}{70} = \frac{1000}{7}\).
The equation is therefore
\(x^2 = \frac{1000}{7}y.\)
To find the length of the cable 30 m from the center, we let \( x = 30 \):

\[
30^2 = \frac{1000}{7} y \quad \text{and} \quad y = \frac{6300}{1000} = 6.3.
\]

So the length of the cable is \( 20 + 6.3 = 26.3 \text{ m} \).

38. Place the parabola with vertex at the origin and axis along the \( x \)-axis. From the given information the point \((12.0,10.0)\) lies on the curve. Substituting coordinates,

\[
y^2 = 4px
\]

\[
(10.0)^2 = 4p(12.0)
\]

\[4p = \frac{100}{12} \quad \text{and} \quad p = 2.08 \text{ cm}
\]

So the light bulb is placed at the focus, 2.08 cm from the vertex.

39.

The required minimum clearance of 12 ft yields the point \((20, -13)\) in the figure.

\[
x^2 = 4py
\]

\[
20^2 = 4p(-13) \quad \text{or} \quad 4p = \frac{20^2}{-13}
\]

Equation: \( x^2 = -\frac{20^2}{13} y \).

When \( y = -25 \),

\[
x^2 = \frac{20^2}{13}(-25)
\]

\[
x = \sqrt{\frac{20^2 \cdot 25}{13}} = \frac{20 \cdot 5}{\sqrt{13}} = \frac{100}{\sqrt{13}}
\]

\[
2x = \frac{200}{\sqrt{13}} \approx 55.5 \text{ ft}
\]

40.

The problem is to find \( y \) in the figure; observe that \( y + 10.0 \) corresponds to the maximum clearance. There are two equations from the form \( x^2 = 4py \):

\[
(30.0)^2 = 4py
\]

\[
(20.0)^2 = 4p(y + 10.0)
\]

\[
(30.0)^2 - (20.0)^2 = -4p(10.0) \quad \text{(subtracting)}
\]

\[
4p = -50
\]
Equation: \(x^2 = -50.0y\); to find \(y\), let \(x = 30.0\):

\[
(30.0)^2 = -50y
\]

\[
y = \frac{30.0^2}{-50.0} = -18.0
\]

So the height of the arch is 18.0 m.

41. We place the vertex of the parabola at the origin, so that one point on the parabola is \((3, -3)\) (from the given dimensions). Substituting in the equation \(x^2 = 4py\), we get

\[
3^2 = 4p(-3)
\]

\[
4p = -3.
\]

The equation is therefore seen to be \(x^2 = -3y\).

![Diagram of a parabola with vertex at the origin, showing the right end of the beam 2 m above the base at \((x, -1)\) and the point \((3, -3)\) on the curve.](image)

The right end of the beam 2 m above the base is at \((x, -1)\). To find \(x\), let \(y = -1\):

\[
x^2 = -3(-1) = 3
\]

\[
x = \pm \sqrt{3}.
\]

Hence the length of the beam is \(2|x| = 2\sqrt{3}\) m.

42. \(y = 1.5x - 0.1x^2 = 0\) when \(x = 15\) ft.

43. \(x^2 = 4py\). From Figure 1.55, we see that the point \((4, 1)\) lies on the curve: \(4^2 = 4p(1)\). So \(p = 4\) ft.

44. Place the parabola with vertex at the origin and axis along the \(x\)-axis. From the given information the point \((1.7, y)\) lies on the curve. The problem is to find \(y\), given that \(p = 11.9\).

\[
y^2 = 4px = 4 \cdot 11.9 \cdot 1.7
\]

Diameter = \(2y = 2\sqrt{4 \cdot 11.9 \cdot 1.7} = 18.0\) in.

1.9 The Ellipse

1. The equation is

\[
\frac{x^2}{25} + \frac{y^2}{16} = 1.
\]

So by (1.16), \(a^2 = 25\) and \(b^2 = 16\); thus \(a = 5\) and \(b = 4\). Since the major axis is horizontal, the vertices are at \((\pm5, 0)\). From \(b^2 = a^2 - c^2\),

\[
16 = 25 - c^2
\]

\[
c^2 = 9
\]

\[
c = \pm 3.
\]

The foci are therefore at \((\pm3, 0)\), on the major axis. Finally, the length of the semi-minor axis is equal to \(b = 4\).
2. \( \frac{x^2}{16} + \frac{y^2}{9} = 1 \)

By (1.16), \( a^2 = 16 \) and \( b^2 = 9 \); so \( a = 4 \) and \( b = 3 \). Since the major axis is horizontal, the vertices are at \((\pm 4,0)\). From \( b^2 = a^2 - c^2 \),
\[
9 = 16 - c^2 \\
c^2 = 7 \\
c = \pm \sqrt{7}
\]
So the foci are at \((\pm \sqrt{7},0)\), on the major axis. Finally, the length of the semiminor axis is \( b = 3 \).

3. The equation is
\[
\frac{x^2}{9} + \frac{y^2}{4} = 1.
\]
So by (1.16), \( a^2 = 9 \) and \( b^2 = 4 \); thus \( a = 3 \) and \( b = 2 \). Since the major axis is horizontal, the vertices are at \((\pm 3,0)\). From \( b^2 = a^2 - c^2 \),
\[
4 = 9 - c^2 \\
c^2 = 5 \\
c = \pm \sqrt{5}.
\]
The foci are therefore at \((\pm \sqrt{5},0)\), on the major axis. Finally, the length of the semi-minor axis is equal to \( b = 2 \).
4. \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)

By (1.17), \( a = 3 \) and \( b = 2 \). From \( b^2 = a^2 - c^2 \),

\[
\begin{align*}
4 &= 9 - c^2 \\
\frac{a^2}{c^2} &= 5 \\
c &= \pm\sqrt{5}
\end{align*}
\]

Since the major axis is vertical, the vertices are at \((0, \pm 3)\) and the foci are at \((0, \pm \sqrt{5})\). The length of the semiminor axis is \( b = 2 \).

5. The equation is

\[
\frac{x^2}{16} + y^2 = 1.
\]

By (1.16), \( a = 4 \) and \( b = 1 \). From \( b^2 = a^2 - c^2 \),

\[
\begin{align*}
1 &= 16 - c^2 \\
\frac{a^2}{c^2} &= 15 \\
c &= \pm\sqrt{15}
\end{align*}
\]

Since the major axis is horizontal, the vertices and foci lie on the \( x \)-axis. The vertices are therefore at \((\pm 4, 0)\) and the foci are at \((\pm\sqrt{15}, 0)\). The length of the semi-minor axis is \( b = 1 \).

6. \( \frac{x^2}{2} + \frac{y^2}{4} = 1 \)

By (1.17), \( a = 2 \) and \( b = \sqrt{2} \), major axis vertical.

\[
\begin{align*}
\frac{a^2}{c^2} &= a^2 - c^2 \\
2 &= 4 - c^2 \\
c &= \pm\sqrt{2}
\end{align*}
\]
Vertices: \((0, \pm 2)\), foci: \((0, \pm \sqrt{2})\), semiminor axis: \(\sqrt{2}\).

7. \[16x^2 + 9y^2 = 144\]
   \[
   \frac{x^2}{9} + \frac{y^2}{16} = 1
   \]
   So by (1.17), \(a^2 = 16\) and \(b^2 = 9\); so \(a = 4\) and \(b = 3\). Since the major axis is vertical, the vertices are at \((0, \pm 4)\). From \(b^2 = a^2 - c^2\)
   \[
   9 = 16 - c^2
   \]
   \[
   c = \pm \sqrt{7}.
   \]
   The foci are therefore at \((0, \pm \sqrt{7})\). Semi-minor axis: \(b = 3\).

![Graph of an ellipse with major axis vertical.

8. \[x^2 + 2y^2 = 4\]
   \[
   \frac{x^2}{4} + \frac{y^2}{2} = 1
   \]
   By (1.16), \(a = 2\) and \(b = \sqrt{2}\), major axis horizontal
   \[
   b^2 = a^2 - c^2
   \]
   \[
   2 = 4 - c^2
   \]
   \[
   c = \pm \sqrt{2}
   \]
   Vertices: \((\pm 2, 0)\), foci: \((\pm \sqrt{2}, 0)\), semiminor axis: \(\sqrt{2}\).

![Graph of an ellipse with major axis horizontal.

9. \[5x^2 + 2y^2 = 20\]
   \[
   \frac{5x^2}{20} + \frac{2y^2}{20} = 1
   \]
   \[
   \frac{x^2}{4} + \frac{y^2}{10} = 1
   \]
   By (1.17), \(a = \sqrt{10}\) and \(b = 2\). Since the major axis is vertical, the vertices are at \((0, \pm \sqrt{10})\).
   From \(b^2 = a^2 - c^2\)
   \[
   4 = 10 - c^2
   \]
   \[
   c = \pm \sqrt{6}.
   \]
   The foci, also on the major axis, are therefore at \((0, \pm \sqrt{6})\), while the length of the semi-minor axis is \(b = 2\). (See sketch in answer section of book.)
10. \[5x^2 + 9y^2 = 45\]
\[\frac{x^2}{9} + \frac{y^2}{5} = 1\]

By (1.16), \(a = 3\) and \(b = \sqrt{5}\), major axis horizontal

\[b^2 = a^2 - c^2\]
\[5 = 9 - c^2\]
\[c = \pm 2\]

Vertices: \((\pm 3,0)\), foci: \((\pm 2,0)\), semimajor axis: \(\sqrt{5}\).

11. \[5x^2 + y^2 = 5\]
\[\frac{x^2}{1} + \frac{y^2}{5} = 1\]

major axis vertical

Vertices: \((0, \pm \sqrt{5})\), foci: \((0, \pm 2)\). Length of semi-minor axis: \(b = 1\). (See sketch in answer section of book.)

12. \[x^2 + 4y^2 = 4\]
\[\frac{x^2}{4} + \frac{y^2}{1} = 1\]

\(a = 2\) and \(b = 1\), major axis horizontal

\[b^2 = a^2 - c^2\]
\[1 = 4 - c^2\]
\[c = \pm \sqrt{3}\]

Vertices: \((\pm 2,0)\), foci: \((\pm \sqrt{3},0)\), semiminor axis: \(1\).

13. \[x^2 + 2y^2 = 6\]
\[\frac{x^2}{6} + \frac{2y^2}{6} = 1\]
\[\frac{x^2}{6} + \frac{y^2}{3} = 1\]

major axis horizontal

Thus \(a = \sqrt{6}\) and \(b = \sqrt{3}\). From \(b^2 = a^2 - c^2\),
\[3 = 6 - c^2\]
\[c = \pm \sqrt{3}\]

Vertices: \((\pm \sqrt{6},0)\); foci: \((\pm \sqrt{3},0)\). Length of semi-minor axis: \(b = \sqrt{3}\). (See sketch in answer section of book.)

14. \[9x^2 + 2y^2 = 18\]
\[\frac{x^2}{2} + \frac{y^2}{9} = 1\]

\(a = 3\) and \(b = \sqrt{2}\), major axis vertical

\[b^2 = a^2 - c^2\]
\[2 = 9 - c^2\]
\[c = \pm \sqrt{7}\]

Vertices: \((0, \pm 3)\), foci: \((0, \pm \sqrt{7})\), semiminor axis: \(\sqrt{2}\).
15. \(15x^2 + 7y^2 = 105\)
\[
\frac{x^2}{7} + \frac{y^2}{15} = 1 \quad \text{major axis vertical}
\]
Thus \(a = \sqrt{15}\) and \(b = \sqrt{7}\). From \(b^2 = a^2 - c^2\)
\[
7 = 15 - c^2
\]
\(c = \pm \sqrt{8} = \pm 2\sqrt{2}.
\]
Vertices: \((0, \pm \sqrt{15})\), foci: \((0, \pm 2\sqrt{2})\). Length of semi-minor axis: \(b = \sqrt{7}\).

16. \(9x^2 + y^2 = 27\)
\[
\frac{x^2}{3} + \frac{y^2}{27} = 1 \quad a = \sqrt{27} = 3\sqrt{3} \text{ and } b = \sqrt{3}, \text{ major axis vertical}
\]
\[
b^2 = a^2 - c^2
\]
\[
3 = 27 - c^2
\]
\(c = \pm \sqrt{24} = \pm 2\sqrt{6}.
\]
Vertices: \((0, \pm 3\sqrt{3})\), foci: \((0, \pm 2\sqrt{6})\), semiminor axis: \(\sqrt{3}\).

17. \(3x^2 + 4y^2 = 12\)
\[
\frac{x^2}{4} + \frac{y^2}{3} = 1 \quad \text{dividing by 12}
\]
Since \(a^2 = 4\), \(a = \pm 2\), so the vertices are at \((\pm 2, 0)\). From \(b^2 = 3\), we get \(b = \sqrt{3}\) (semiminor axis). \(b^2 = a^2 - c^2\) yields \(c = \pm 1\).

18. \(\frac{x^2}{5} + \frac{y^2}{12} = 1\) yields \(a = \pm \sqrt{5}, b = \pm 2\sqrt{3}\).

From \(b^2 = a^2 - c^2\), we get \(c = \pm \sqrt{7}\).

20. Graph the two equations \(y = \sqrt{\frac{1}{2}(5 - 6x^2)}\) and \(y = -\sqrt{\frac{1}{2}(5 - 6x^2)}\).

22. Graph the two equations \(y = \pm \sqrt{1 - \frac{1}{4}x^2}\).

23. Since the foci \((\pm 2, 0)\) lie along the major axis, the major axis is horizontal. So the form of the equation is
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.
\]
Since the vertices are at \((\pm 3, 0)\), \(a = 3\). From \(b^2 = a^2 - c^2\) (with \(c = 2\)), we get \(b^2 = 9 - 4 = 5\).
So the equation is
\[
\frac{x^2}{9} + \frac{y^2}{5} = 1.
\]

24. Since the foci are at \((0, \pm 3)\), the major axis is vertical
\[
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{(form)}
\]
Since the length of the major axis is 8, \(a = 4\), while \(c = 3\). Hence \(b^2 = a^2 - c^2 = 16 - 9 = 7\)
and \(\frac{x^2}{7} + \frac{y^2}{16} = 1\).
25. Since the foci \((0, \pm 2)\) lie on the major axis, the major axis is vertical. So by (1.17) the form of the equation is
\[
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.
\]
Since the length of the major axis is 8, \(a = 4\). From \(b^2 = a^2 - c^2\) with \(c = 2\), \(b^2 = 16 - 4 = 12\). Hence
\[
\frac{x^2}{12} + \frac{y^2}{16} = 1 \quad \text{or} \quad 4x^2 + 3y^2 = 48.
\]

26. Since the length of the entire major axis is 6, we get \(a = 3\). Foci at \((0, \pm 2)\) implies that \(c = 2\) and that the major axis is vertical. From \(b^2 = a^2 - c^2\) and from the form
\[
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{we get} \quad \frac{x^2}{5} + \frac{y^2}{9} = 1.
\]

27. Since the foci are at \((0, \pm 3)\), \(c = 3\), and the major axis is vertical. By (1.17)
\[
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.
\]
Since the length of the minor axis is 6, \(b = 3\). From \(b^2 = a^2 - c^2\), \(9 = a^2 - 9\) and \(a^2 = 18\).
Equation: \(\frac{x^2}{9} + \frac{y^2}{18} = 1\) or \(2x^2 + y^2 = 18\).

28. Since the length of the entire minor axis is 4, \(b = 2\). Foci at \((0, \pm 2)\) implies that \(c = 2\) and that the major axis is vertical. From \(b^2 = a^2 - c^2\), \(4 = a^2 - 4\) and \(a^2 = 8\). By (1.17)
\[
\frac{x^2}{4} + \frac{y^2}{8} = 1.
\]

29. Since the vertices and foci are on the \(y\)-axis, the form of the equation is, by (1.17),
\[
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.
\]
From \(b^2 = a^2 - c^2\) with \(a = 8\) and \(c = 5\), \(b^2 = 64 - 25 = 39\). Hence
\[
\frac{x^2}{39} + \frac{y^2}{64} = 1.
\]

30. Foci at \((\pm 3, 0)\) implies that \(c = 3\) and that the major axis is horizontal; \(b = 4\) (seminor axis). From \(b^2 = a^2 - c^2\), we have \(16 = a^2 - 9\), or \(a^2 = 25\). By (1.16)
\[
\frac{x^2}{25} + \frac{y^2}{16} = 1 \quad \text{or} \quad 16x^2 + 25y^2 = 400.
\]

31. Form: \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\); \(c = 2\sqrt{3}\), \(b = 2\). From \(b^2 = a^2 - c^2\), \(4 = a^2 - (2\sqrt{3})^2\) and \(a^2 = 16\).
Equation: \(\frac{x^2}{16} + \frac{y^2}{4} = 1\) or \(x^2 + 4y^2 = 16\).

32. Since the foci and vertices lie on the major axis, the major axis is horizontal. Moreover, \(c = \sqrt{5}\) and \(a = \sqrt{7}\). So \(b^2 = 7 - 5 = 2\) and by (1.16),
\[
\frac{x^2}{7} + \frac{y^2}{2} = 1 \quad \text{or} \quad 2x^2 + 7y^2 = 14.
\]
33. From the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)
we get (since \( b = 2 \))
\[ \frac{x^2}{a^2} + \frac{y^2}{4} = 1 \]
Substituting the coordinates of the point \((3, 1)\) yields
\[ \frac{9}{a^2} + \frac{1}{4} = 1 \quad \text{and} \quad \frac{9}{a^2} = \frac{3}{4}. \]
So
\[ \frac{a^2}{9} = \frac{4}{3} \quad \text{and} \quad a^2 = 12. \]
Equation:
\[ \frac{x^2}{12} + \frac{y^2}{4} = 1 \quad \text{or} \quad x^2 + 3y^2 = 12 \]

34. Since \( b = 3 \), we get from (1.17), \( \frac{x^2}{9} + \frac{y^2}{a^2} = 1. \) Since (1,4) lies on the curve, the values \( x = 1 \) and \( y = 4 \) satisfies the equation:
\[ \frac{1^2}{9} + \frac{4^2}{a^2} = 1 \quad \text{whence} \quad a^2 = 18 \quad \text{and} \quad 2x^2 + y^2 = 18. \]

35. From the original derivation of the ellipse, \( 2a = 16 \) and \( a = 8. \) Since the foci are at \((\pm 6, 0)\),
\( c = 6. \) Thus \( b^2 = a^2 - c^2 = 64 - 36 = 28. \)
By (1.16) the equation is
\[ \frac{x^2}{64} + \frac{y^2}{28} = 1. \]

36. Vertices at \((\pm 4, 0)\) tell us that \( a = 4 \) and that the major axis is horizontal. The definition of eccentricity gives the following equation:
\[ e = \frac{1}{2} = \frac{c}{a} \quad \text{or} \quad \frac{1}{2} = \frac{c}{4} \]
which yields \( c = 2. \) Finally, \( b^2 = a^2 - c^2 = 16 - 4 = 12. \) By (1.16),
\[ \frac{x^2}{16} + \frac{y^2}{12} = 1 \quad \text{or} \quad 3x^2 + 4y^2 = 48. \]

37. \( 9x^2 + 5y^2 = 45 \) or \( \frac{x^2}{5} + \frac{y^2}{9} = 1; \ a = 3; \ b = \sqrt{5}. \) From \( b^2 = a^2 - c^2, \ 5 = 9 - c^2 \) and \( c = 2. \) Thus
\[ e = \frac{c}{a} = \frac{2}{3}. \]

38. Distance from \((x, y)\) to \((0, 0)\): \[ \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}. \]
Distance from \((x, y)\) to \((3, 0)\): \[ \sqrt{(x - 3)^2 + (y - 0)^2} = \sqrt{(x - 3)^2 + y^2}. \]
From the given condition:
\[ \sqrt{x^2 + y^2} = 2\sqrt{(x - 3)^2 + y^2} \]
\[ x^2 + y^2 = 4[(x - 3)^2 + y^2] \quad \text{squaring both sides} \]
\[ x^2 + y^2 = 4(x^2 - 6x + 9 + y^2) \]
\[ x^2 + y^2 = 4x^2 - 24x + 36 + 4y^2 \]
\[ 0 = 3x^2 - 24x + 36 + 3y^2 \]
\[ 3x^2 + 3y^2 - 24x + 36 = 0 \]
\[ x^2 + y^2 - 8x + 12 = 0. \quad \text{The locus is a circle.} \]
39. We want the center of the ellipse to be at the origin with the center of the earth at one of the foci. Study the following diagram:

\[ b^2 = 4100^2 - 20^2 = 16,809,600. \]

40. Let \( A \) = the maximum distance and \( P \) = the minimum distance, as shown.

\[ A \] is also the distance from the left focus to the right vertex. So \( A - P \) is the distance between foci. Therefore \( \frac{1}{2}(A - P) \) is the distance from the center to the sun (the focus), or \( c = \frac{1}{2}(A - P) \).

Now \( a = c + P = \frac{1}{2}(A - P) + P = \frac{1}{2}(A + P) \) so

\[ e = \frac{c}{a} = \frac{\frac{1}{2}(A - P)}{\frac{1}{2}(A + P)} = \frac{A - P}{A + P}. \]

In our problem \( e = \frac{9.46 \times 10^7 - 9.14 \times 10^7}{9.47 \times 10^7 + 9.14 \times 10^7} = 0.0172 \approx \frac{1}{60} \).

41. Let \( A \) = the maximum distance and \( P \) = the minimum distance as shown.

\( A \) is also the distance from the left focus to the right vertex. So \( A - P \) is the distance between the foci. Therefore \( \frac{1}{2}(A - P) \) is the distance from the center to the sun (the focus), or \( c = \frac{1}{2}(A - P) \).

Now \( a = c + P = \frac{1}{2}(A - P) + P = \frac{1}{2}(A + P) \). So

\[ e = \frac{c}{a} = \frac{\frac{1}{2}(A - P)}{\frac{1}{2}(A + P)} = \frac{A - P}{A + P}. \]
In our problem
\[ e = \frac{3.285 \times 10^9 - 5.48 \times 10^7}{3.285 \times 10^9 + 5.48 \times 10^7} = 0.967. \]

42. \[ A = \pi \cdot a \cdot b = \pi \cdot 15 \cdot 12 \approx 565 \text{ ft}^2 \]

43. Since \( a = 2 \) and \( b = \frac{3}{2} \), we get
\[ \frac{x^2}{4} + \frac{y^2}{\frac{9}{4}} = 1 \quad \text{or} \quad \frac{x^2}{4} + \frac{4y^2}{9} = 1, \]

and \( 9x^2 + 16y^2 = 36. \)

44. From the figure, \( a = 15 \) and \( b = 5. \) From (1.17)
\[ \frac{x^2}{5^2} + \frac{y^2}{15^2} = 1. \]

The next step is to find \( x \) when \( y = 10: \)
\[ \frac{x^2}{25} = \frac{225}{225} - \frac{100}{225} = \frac{125}{225} \quad \text{and} \quad x^2 = \frac{5 \cdot 25 \cdot 25}{15^2} \]
So \( x = \frac{\sqrt{5} \cdot 25}{15} \) and the length of the beam is \( 2x = 10 \frac{\sqrt{5}}{3} \approx 7.5 \text{ ft} \)

![Graph of an ellipse](image)

45. Placing the center at the origin, the vertices are at \((\pm 6, 0)\). The road extends from \((-4, 0)\) to \((4, 0)\). Since the clearance is \(4 \text{ m}\), the point \((4, 4)\) lies on the ellipse, as shown.

![Graph of an ellipse](image)

By (1.16),
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]
\[ \frac{x^2}{36} + \frac{y^2}{b^2} = 1. \]

To find \( b \), we substitute the coordinates of \((4, 4)\) in the equation:
\[ \frac{16}{36} + \frac{16}{b^2} = 1 \]
\[ \frac{16}{b^2} = \frac{36 - 16}{36} = \frac{20}{36} = \frac{5}{9} \]
\[ b^2 = \frac{5}{9} \frac{(9)(16)}{(3)(4)} = 12 \frac{12\sqrt{5}}{5}. \]
So the height of the arch is \( \frac{12\sqrt{5}}{5} \approx 5.4 \text{ m} \) to two significant digits.

46. Since the sun is at one of the foci, \( a - c = 12 \) (in millions of miles) when the comet is closest to the sun. From \( e = \frac{c}{a} \), we get

\[
0.99992 = \frac{c}{a} \quad \text{or} \quad c = 0.99992a.
\]

So \( a - c = a - 0.99992a = 12 \); solving, \( a = 150000 \) and \( c = 149988 \).

Farthest point: \((150000 + 149988)(100000) = 3.0 \times 10^{11} \text{ mi.}\)

### 1.10 The Hyperbola

1. Comparing the given equation,

\[
\frac{x^2}{16} - \frac{y^2}{9} = 1
\]

to form (1.22), we see that the transverse axis is horizontal, with \( a^2 = 16 \) and \( b^2 = 9 \). So \( a = 4 \) and \( b = 3 \). From \( b^2 = c^2 - a^2 \), we get

\[
\begin{align*}
9 & = c^2 - 16 \\
c & = \pm 5.
\end{align*}
\]

So the vertices are at \((\pm 4, 0)\) and the foci are at \((\pm 5, 0)\). Using \( a = 4 \) and \( b = 3 \), we draw the auxiliary rectangle and sketch the curve:

![Auxiliary rectangle and sketch of hyperbola](image)

2. Comparing the given equation \( \frac{x^2}{9} - \frac{y^2}{4} = 1 \) to the form (1.22), we see that the transverse axis is horizontal, with \( a^2 = 9 \) and \( b^2 = 4 \). So \( a = 3 \) and \( b = 2 \). From \( b^2 = c^2 - a^2 \), we get \( 4 = c^2 - 9 \) and \( c^2 = 13 \). It follows that the vertices are at \((\pm 3, 0)\) and the foci at \((\pm \sqrt{13}, 0)\). Using \( a = 3 \) and \( b = 2 \), we draw the auxiliary rectangle and sketch the curve.

![Auxiliary rectangle and sketch of hyperbola](image)

3. \( \frac{x^2}{9} - \frac{y^2}{16} = 1 \); by Equation (1.22), the transverse axis is horizontal with \( a^2 = 9 \) and \( b^2 = 16 \).

So \( a = 3 \) and \( b = 4 \). From \( b^2 = c^2 - a^2 \), we have \( 16 = c^2 - 9 \) or \( c = \pm 5 \).
1.10. THE HYPERBOLA

It follows that the vertices are at \((\pm 3, 0)\) and the foci are at \((\pm 5, 0)\). Using \(a = 3\) and \(b = 4\), we draw the auxiliary rectangle and the asymptotes, and then sketch the curve.

4. Equation: \(\frac{x^2}{16} - \frac{y^2}{4} = 1\)

By (1.22), \(a = 4\) and \(b = 2\), transverse axis horizontal. From \(b^2 = c^2 - a^2\), \(4 = c^2 - 16\) and \(c = \pm\sqrt{20} = \pm 2\sqrt{5}\). So the vertices are at \((\pm 4, 0)\) and the foci at \((\pm 2\sqrt{5}, 0)\). Using \(a = 4\) and \(b = 2\), we draw the auxiliary rectangle and sketch the curve.

5. By (1.23), \(a = 2\) and \(b = 2\), transverse axis vertical along the \(y\)-axis. From \(b^2 = c^2 - a^2\), \(4 = c^2 - 4\) and \(c = \pm\sqrt{8} = \pm 2\sqrt{2}\). So the vertices are at \((0, \pm 2)\) and the foci at \((0, \pm 2\sqrt{2})\). Using \(a = 2\) and \(b = 2\), we draw the auxiliary rectangle and sketch the curve:

6. Equation: \(\frac{y^2}{4} - \frac{x^2}{8} = 1\)

By (1.23), \(a = 2\) and \(b = \sqrt{8} = 2\sqrt{2}\), transverse axis vertical. From \(b^2 = c^2 - a^2\), \(8 = c^2 - 4\) and \(c = \pm\sqrt{12} = \pm 2\sqrt{3}\). So the vertices are at \((0, \pm 2)\) and the foci at \((0, \pm 2\sqrt{3})\). Using \(a = 2\) and \(b = 2\sqrt{2}\), we draw the auxiliary rectangle and sketch the curve.
7. \( x^2 - \frac{y^2}{5} = 1 \) transverse axis horizontal

\( a^2 = 1 \) and \( b^2 = 5 \); so \( a = 1 \) and \( b = \sqrt{5} \). From \( b^2 = c^2 - a^2 \), \( 5 = c^2 - 1 \) and \( c = \pm \sqrt{6} \). Vertices: \((\pm 1, 0)\); foci: \((\pm \sqrt{6}, 0)\). Using \( a = 1 \) and \( b = \sqrt{5} \), we draw the auxiliary rectangle and sketch the curve.

8. Equation: \( 9y^2 - 2x^2 = 18 \) or \( \frac{y^2}{2} - \frac{x^2}{9} = 1 \)

By (1.23), \( a = \sqrt{2} \) and \( b = 3 \), transverse axis vertical. From \( b^2 = c^2 - a^2 \), \( 9 = c^2 - 2 \) and \( c = \pm \sqrt{11} \). So the vertices are at \((0, \pm \sqrt{2})\) and the foci at \((0, \pm \sqrt{11})\). Using \( a = \sqrt{2} \) and \( b = 3 \), we draw the auxiliary rectangle and sketch the curve.

9. \( 2y^2 - 3x^2 = 24 \)

\( \frac{2y^2}{24} - \frac{3x^2}{24} = 1 \)

\( \frac{y^2}{12} - \frac{x^2}{8} = 1 \)

By (1.23), \( a = \sqrt{12} = 2\sqrt{3} \) and \( b = \sqrt{8} = 2\sqrt{2} \). From \( b^2 = c^2 - a^2 \), \( 8 = c^2 - 12 \), so that \( c = \pm \sqrt{20} = \pm 2\sqrt{5} \). Since the transverse axis lies along the \( y \)-axis, the vertices are at \((0, \pm 2\sqrt{3})\) and the foci at \((0, \pm 2\sqrt{5})\). Using \( a = 2\sqrt{3} \) and \( b = 2\sqrt{2} \), we draw the auxiliary rectangle and sketch the curve.
10. Equation: $\frac{x^2}{6} - \frac{y^2}{6} = 1$

By (1.22), $a = \sqrt{6}$ and $b = \sqrt{6}$, transverse axis horizontal. From $b^2 = c^2 - a^2$, $6 = c^2 - 6$ and $c = \pm\sqrt{12} = \pm2\sqrt{3}$. So the vertices are at $(\pm\sqrt{6}, 0)$ and the foci at $(\pm2\sqrt{3}, 0)$. Using $a = \sqrt{6}$ and $b = \sqrt{6}$, we draw the auxiliary rectangle and sketch the curve.

11. $3y^2 - 2x^2 = 6$ or $\frac{y^2}{2} - \frac{x^2}{3} = 1$

By (1.23) the transverse axis is vertical with $a = \sqrt{2}$ and $b = \sqrt{3}$. From $b^2 = c^2 - a^2$, $3 = c^2 - 2$ or $c = \pm\sqrt{5}$.

So the vertices are at $(0, \pm\sqrt{2})$ and the foci at $(0, \pm\sqrt{3})$. Using $a = \sqrt{2}$ and $b = \sqrt{3}$, we draw the auxiliary rectangle and sketch the curve.

12. Equation: $11x^2 - 7y^2 = 77$ or $\frac{x^2}{7} - \frac{y^2}{11} = 1$

By (1.22), $a = \sqrt{7}$ and $b = \sqrt{11}$, transverse axis horizontal. From $b^2 = c^2 - a^2$, $11 = c^2 - 7$ and $c = \pm\sqrt{18} = \pm3\sqrt{2}$. So the vertices are at $(\pm\sqrt{7}, 0)$ and the foci at $(\pm3\sqrt{2}, 0)$. Using $a = \sqrt{7}$ and $b = \sqrt{11}$, we draw the auxiliary rectangle and sketch the curve.
13. Since the foci (and hence the vertices) lie on the $x$-axis, the transverse axis is horizontal. By (1.22),
\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \]
Since the length of the transverse axis is 4, $a = 2$, and since the length of the conjugate axis is 2, $b = 1$. It follows that
\[ \frac{x^2}{4} - \frac{y^2}{1} = 1 \quad \text{and} \quad x^2 - 4y^2 = 4. \]

14. Since the foci (and hence the vertices) lie on the $y$-axis, the transverse axis is vertical. By (1.23), \[ \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1. \] Since the length of the transverse axis is 4, $a = 2$, and since the length of the conjugate axis is 8, $b = 4$. It follows that
\[ \frac{y^2}{4} - \frac{x^2}{16} = 1 \quad \text{or} \quad 4y^2 - x^2 = 16. \]

15. Since the foci (and hence the vertices) lie on the $y$-axis, the form is
\[ \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1. \]
Since the length of the transverse axis is 8, $a = 4$, while $c = 6$. From $b^2 = c^2 - a^2$, we get $b^2 = 36 - 16 = 20$. So the equation is
\[ \frac{y^2}{16} - \frac{x^2}{20} = 1. \]

16. Since the foci (and hence the vertices) lie on the $x$-axis, the transverse axis is horizontal and the form is
\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \]
Since the length of the transverse axis is 6, $a = 3$, while $c = 4$. From $b^2 = c^2 - a^2$, we get $b^2 = 16 - 9 = 7$. So the equation is
\[ \frac{x^2}{9} - \frac{y^2}{7} = 1 \quad \text{or} \quad 7x^2 - 9y^2 = 63. \]

17. Since the vertices lie on the $x$-axis, the form is
\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \]
The conjugate axis of length 8 implies that $b = 4$ and the position of the vertices imply that $a = 4$.
\[ \text{Equation:} \quad \frac{x^2}{16} - \frac{y^2}{16} = 1 \quad \text{or} \quad x^2 - y^2 = 16. \]

18. Since the vertices are on the $y$-axis, the form is
\[ \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1. \]
Since $a = 5$ and $b = 6$, the equation is
\[ \frac{y^2}{25} - \frac{x^2}{36} = 1. \]
19. Since the vertices are on the $y$-axis, the form is

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$ 

Since $a = 5$ and $c = 7$, we get $b^2 = c^2 - a^2 = 49 - 25 = 24$. Thus

$$\frac{y^2}{25} - \frac{x^2}{24} = 1.$$ 

20. Since the foci lie on the $y$-axis, the transverse axis is vertical. By (1.23)

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$ 

Since the length of the conjugate axis is 8, $b = 4$. The foci are at $(0, \pm 5)$, so that $c = 5$. From $b^2 = c^2 - a^2$, we have $16 = 25 - a^2$ and $a^2 = 9$. 

Equation: $\frac{y^2}{9} - \frac{x^2}{16} = 1$ or $16y^2 - 9x^2 = 144$.

21. Since the foci are on the $x$-axis, the form is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$ 

Since the length of the conjugate axis is 10, $b = 5$, while $c = 6$. From $b^2 = c^2 - a^2$, we get $a^2 = c^2 - b^2 = 36 - 25 = 11$. So the equation is

$$\frac{x^2}{11} - \frac{y^2}{25} = 1.$$ 

22. Start with Eq. (1.19):

$$\sqrt{(x - c)^2 + y^2} - \sqrt{(x + c)^2 + y^2} = \pm 2a$$

$$\sqrt{(x - c)^2 + y^2} = \pm 2a + \sqrt{(x + c)^2 + y^2}$$

isolating the radical

$$(x - c)^2 + y^2 = 4a^2 \pm 4a\sqrt{(x + c)^2 + y^2} + (x + c)^2 + y^2$$

squaring both sides

$$x^2 - 2cx + c^2 + y^2 = 4a^2 \pm 4a\sqrt{(x + c)^2 + y^2} + x^2 + 2cx + c^2 + y^2$$

$$-4cx - 4a^2 = \pm 4a\sqrt{(x + c)^2 + y^2}$$

collecting terms

$$cx + a^2 = \pm a\sqrt{(x + c)^2 + y^2}$$

$$c^2x^2 + 2ca^2x + a^4 = a^2(x^2 + 2cx + c^2 + y^2)$$

squaring both sides

$$c^2x^2 + 2ca^2x + a^4 = a^2x^2 + 2ca^2x + a^2c^2 + a^2y^2$$

$$c^2x^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4$$

collecting terms

$$(c^2 - a^2) x^2 - a^2y^2 = a^2(c^2 - a^2)$$

factoring

$$\frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1$$

dividing by $a^2(c^2 - a^2)$

23. By the original derivation of the equation of the hyperbola, $2a = 6$ and $a = 3$. Since $(0, \pm 5)$ are the foci, $c = 5$. Thus $b^2 = 25 - 9 = 16$. By (1.23)

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1 \text{ or } 16y^2 - 9x^2 = 144.$$
24. Since the foci are at \((\pm 3,0)\), we know that \(c = 3\) and the transverse axis is horizontal. From Eq. (1.21), \(y = \pm \frac{b}{a} x = \pm \frac{4}{3} x\) and \(\frac{b}{a} = \frac{4}{3}\). So we get \(b = \frac{4}{3}a\); from \(b^2 = c^2 - a^2\), we get
\[
\frac{16}{9} a^2 = 9 - a^2 \\
\frac{25}{9} a^2 = 9 \\
\frac{a^2}{81/25} = \frac{81}{9/25} \quad \text{and} \quad b^2 = \frac{16}{9} \cdot \frac{81}{25} = \frac{9 \cdot 16}{25} = \frac{144}{25}.
\]
By (1.22), \(\frac{y^2}{81/25} - \frac{x^2}{144/25} = 1\) or \(\frac{25x^2}{81} - \frac{25y^2}{144} = 1\).

25. By (1.23), the equation has the form \(\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1\). Since \(a = 12\), we have
\[
\frac{y^2}{144} - \frac{x^2}{b^2} = 1.
\]
To find \(b\), we substitute the coordinates of \((-1,13)\) in the last equation:
\[
\frac{169}{144} - \frac{1}{b^2} = 1 \\
\frac{1}{b^2} = \frac{144}{144} - \frac{169}{144} = \frac{25}{144}.
\]
Thus \(b^2 = \frac{144}{25}\). The equation is
\[
\frac{y^2}{144} - \frac{x^2}{144/25} = 1 \quad \text{or} \quad \frac{y^2}{144} - \frac{25x^2}{144} = 1.
\]

26.

27. \(pV = k\)
\[(12)(3.0) = k \quad V = 3.0 \, \text{m}^3, \, p = 12 \, \text{Pa}\)
So \(pV = 36\). (See graph in answer section of book.)

28. \(b^2 = c^2 - a^2\)
\[33 = c^2 - 16\]
\[c = \pm 7\]
So the reflected ray crosses the \(x\)-axis at \(x = -7\).

1.11 Translation of Axes; Standard Equations of the Conics

1. Circle, center at \((1,2)\), \(r = \sqrt{3}\).

2. The equation may be viewed as the ellipse \(\frac{x^2}{9} + \frac{y^2}{5} = 1\) rigidly translated from its position with center at the origin to the center at \((1,2)\).
3. \((y + 3)^2 = 8(x - 2)\)
\((y + 3)^2 = 4(2)(x - 2)\) \quad \text{\(p = 2\)}

Vertex at \((2, -3)\), focus at \((2 + 2, -3) = (4, -3)\).

4. The equation may be viewed as the hyperbola \(\frac{x^2}{4} - \frac{y^2}{9} = 1\) rigidly translated from its position with center at the origin to center at \((3, 0)\).

5. \(2x^2 - 3y^2 + 8x - 12y + 14 = 0\)
\(2x^2 + 8x - 3y^2 - 12y + 14 = 0\)
\(2(x^2 + 4x) - 3(y^2 + 4y) = -14\) \quad \text{factoring 2 and -3}

Note that the square of one-half the coefficient of \(x\) and \(y\) is \(\left(\frac{1}{2} \cdot 4\right)^2 = 4\).

Inserting these values inside the parentheses and balancing the equation, we get
\[2(x^2 + 4x + \frac{4}{4}) - 3(y^2 + 4y + \frac{4}{4}) = -14 + 2 \cdot \frac{4}{4} - 3 \cdot \frac{4}{4}\]
\[2(x + 2)^2 - 3(y + 2)^2 = -18\]
\[\frac{3(y + 2)^2}{6} - \frac{2(x + 2)^2}{9} = 1\]
\[\frac{18}{6} - \frac{18}{9} = 1\]

The equation represents a hyperbola with transverse axis vertical. Center: \((-2, -2)\),
\(a = \sqrt{6}, \quad b = 3\).
6. \(4x^2 - 4x - 48y + 193 = 0\)

\[
\begin{align*}
4x^2 - 4x &= 48y - 193 \\
x^2 - x &= 12y - \frac{193}{4} \quad \text{dividing by 4}
\end{align*}
\]

Adding \(\left(\frac{1}{2} - 1\right)\) to each side,

\[
\begin{align*}
x^2 - x + \frac{1}{4} &= 12y - \frac{193}{4} + \frac{1}{4} = 12y - 48 \\
\left(x - \frac{1}{2}\right)^2 &= 12(y - 4) \\
\left(x - \frac{1}{2}\right)^2 &= 4(3)(y - 4) \quad p = 3
\end{align*}
\]

This is the equation of a parabola with vertex at \((\frac{1}{2}, 4)\). Since \(p = 3\), the focus is at \(\left(\frac{1}{2}, 4 + 3\right) = \left(\frac{1}{2}, 7\right)\).

7. \(16x^2 + 4y^2 + 64x - 12y + 57 = 0\)

\[
\begin{align*}
16x^2 + 64x + 4y^2 - 12y + 57 &= 0 \\
16(x^2 + 4x) + 4(y^2 - 3y) &= -57 \quad \text{factoring 16 and 4}
\end{align*}
\]

Note that

\[
\begin{align*}
\left(\frac{1}{2} \cdot 4\right)^2 &= 4 \quad \text{and} \quad \left[\frac{1}{2}(-3)\right]^2 = \frac{9}{4}
\end{align*}
\]

Inserting these values inside the parentheses and balancing the equation, we get

\[
\begin{align*}
16(x^2 + 4x + 4) + 4(y^2 - 3y + \frac{9}{4}) &= -57 + 16 \cdot 4 + 4\left(\frac{9}{4}\right) \\
16(x + 2)^2 + 4(y - \frac{3}{2})^2 &= 16 \\
\frac{(x + 2)^2}{4} + \frac{(y - 3/2)^2}{1} &= 1.
\end{align*}
\]

The equation represents an ellipse with major axis vertical. Center: \((-2, \frac{3}{2})\), \(a = 2\), \(b = 1\).
8. $y^2 - 12y - 5x + 41 = 0$
   $y^2 - 12y = 5x - 41$
   Adding $\left[ \frac{1}{2}(-12) \right]^2 = 36$ to each side,

   
   \[
   y^2 - 12y + 36 = 5x - 41 + 36
   \]

   \[
   (y - 6)^2 = 5x - 5 = 5(x - 1) \quad \text{factoring}
   \]

   \[
   (y - 6)^2 = 4 \cdot \frac{5}{4}(x - 1) \quad p = \frac{5}{4}
   \]

   This is a parabola with vertex at $(1, 6)$ and focus at \( \left( 1 + \frac{5}{4}, 6 \right) = \left( \frac{9}{4}, 6 \right) \).

9. $x^2 + y^2 + 2x - 2y + 2 = 0$
   $x^2 + 2x + y^2 - 2y = -2$
   \[
   (x^2 + 2x + 1) + (y^2 - 2y + 1) = -2 + 1 + 1
   \]

   \[
   (x + 1)^2 + (y - 1)^2 = 0
   \]

   Point: $(-1, 1)$.

10. $x^2 + 2y^2 - 6x + 4y + 1 = 0$
    $x^2 - 6x + 2y^2 + 4y = -1$
    $x^2 - 6x + 2(y^2 + 2y) = -1$ \quad \text{factoring}
    
    Add \( \left[ \frac{1}{2}(-6) \right]^2 = 9 \) and \( \left[ \frac{1}{2}(2) \right]^2 = 1 \) to each side (inside the parentheses):

    \[
    x^2 - 6x + 9 + 2(y^2 + 2y + 1) = -1 + 9 + 2(1)
    \]

    \[
    (x - 3)^2 + 2(y - 1)^2 = 10
    \]

    \[
    \frac{(x - 3)^2}{10} + \frac{(y + 1)^2}{5} = 1
    \]
Ellipse, center at $(3, -1)$ with $a = \sqrt{10}$ and $b = \sqrt{5}$.

11. $2x^2 - 12y^2 + 60y - 63 = 0$
   
   $2x^2 - 12(y^2 - 5y) = 63$

   The square of one-half the coefficient of $y$ is $\left(\frac{1}{2}(-5)\right)^2 = \frac{25}{4}$. Inserting this number inside the parentheses, we get
   
   $2x^2 - 12(y^2 - 5y + \frac{25}{4}) = 63 - 12(\frac{25}{4}) = -12$
   
   $x^2 - 6(y^2 - 5y + \frac{25}{4}) = -6$
   
   $x^2 - 6(y - \frac{5}{2})^2 = -6$
   
   $(y - \frac{5}{2})^2 - \frac{x^2}{6} = 1.$

Hyperbola, center at $(0, \frac{5}{2})$, transverse axis vertical with $a = 1$ and $b = \sqrt{6}$.

12. $2x^2 + 3y^2 - 8x - 18y + 35 = 0$
   
   $2x^2 - 8x + 3y^2 - 18y = -35$
   
   $2(x^2 - 4x ) + 3(y^2 - 6y) = -35$

   Observe that $\left(\frac{1}{2}(-4)\right)^2 = 4$ and $\left(\frac{1}{2}(-6)\right)^2 = 9$. Inserting these values inside the parentheses and balancing the equation, we get

   $2(x^2 - 4x + 4) + 3(y^2 - 6y + 9) = -35 + 2(4) + 3(9)$
   
   $2(x - 2)^2 + 3(y - 3)^2 = 0$

   Single point $(2, 3)$.

13. $64x^2 + 64y^2 - 16x - 96y - 27 = 0$
   
   $64x^2 - 16x + 64y^2 - 96y - 27 = 0$
   
   $64(x^2 - \frac{x}{4}) + 64(y^2 - \frac{3y}{4}) = 27$
   
   $64(x^2 - \frac{x}{4} + \frac{1}{64}) + 64(y^2 - \frac{3y}{2} + \frac{9}{16}) = 27 + 1 + 36$
   
   $64(x - \frac{1}{8})^2 + 64(y - \frac{3}{4})^2 = 64$
   
   $(x - \frac{1}{8})^2 + (y - \frac{3}{4})^2 = 1.$

   Circle of radius 1 centered at $\left(\frac{1}{8}, \frac{3}{4}\right)$.
14. \(4x^2 - 4x - 16y + 5 = 0\)
\[4x^2 - 4x = 16y - 5\]
\[x^2 - x = 4y - \frac{5}{4}\] dividing by 4
Add \(\left[\frac{1}{2}(1)\right]^2 = \frac{1}{4}\) to each side:

\[x^2 - x + \frac{1}{4} = 4y - \frac{5}{4} + \frac{1}{4} = 4y - 1\]
\[
\left(x - \frac{1}{2}\right)^2 = 4\left(y - \frac{1}{4}\right) \quad \text{factoring}
\]
\[
\left(x - \frac{1}{2}\right)^2 = 4(1)\left(y - \frac{1}{4}\right) \quad p = 1
\]
Parabola with vertex at \(\left(\frac{1}{2}, \frac{1}{4}\right)\) and focus at \(\left(\frac{1}{2}, \frac{1}{4} + 1\right) = \left(\frac{1}{2}, \frac{5}{4}\right)\).

15. \(3x^2 + y^2 - 18x + 2y + 29 = 0\)
\[3x^2 - 18x + y^2 + 2y = -29\]
\[3(x^2 - 6x) + (y^2 + 2y) = -29\]
Observe that \(\left[\frac{1}{2}(-6)\right]^2 = 9\) and \(\frac{1}{2} \cdot 2\) = 1. Adding these values inside the parentheses and balancing the equation, we get
\[3(x^2 - 6x + 9) + (y^2 + 2y + 1) = -29 + 3 \cdot 9 + 1\]
\[3(x - 3)^2 + (y + 1)^2 = -1\]
which is an imaginary locus.

16. \(100x^2 - 180x - 100y + 81 = 0\)
\[100x^2 - 180x = 100y - 81\]
\[x^2 - \frac{9}{5}x = y - \frac{81}{100}\]
Add \(\left[\frac{1}{2}(\frac{-9}{5})\right]^2 = \frac{81}{100}\) to each side:

\[x^2 - \frac{9}{5}x + \frac{81}{100} = y\]
\[
\left(x - \frac{9}{10}\right)^2 = 4 \cdot \frac{1}{4} y \quad p = \frac{1}{4}
\]
Parabola, vertex at \(\left(\frac{9}{10}, 0\right)\), focus at \(\left(\frac{9}{10}, \frac{1}{4}\right)\).
17. \( x^2 + 2x - 12y + 25 = 0 \)

\[
x^2 + 2x = 12y - 25
\]

We add to each side of the equation the square of one-half the coefficient of \( x \), \( \left( \frac{1}{2} \cdot 2 \right)^2 = 1 \):

\[
\begin{align*}
x^2 + 2x + 1 &= 12y - 25 + 1 \\
(x + 1)^2 &= 12y - 24 \\
(x + 1)^2 &= 12(y - 2)
\end{align*}
\]

\[
(x + 1)^2 = 4 \cdot 3(y - 2), \quad p = +3
\]

Vertex at \((-1, 2)\), focus at \((-1, 5)\).

18. \( 2x^2 - y^2 - 8x - 2y + 3 = 0 \)

\[
\begin{align*}
2x^2 - 8x - y^2 - 2y &= -3 \\
2(x^2 - 4x) - (y^2 + 2y) &= -3 \quad \text{factoring}
\end{align*}
\]

Observe that \( \left( \frac{1}{2}(-4) \right)^2 = 4 \) and \( \left( \frac{1}{2}(2) \right)^2 = 1 \). Inserting these values inside the parentheses and balancing the equation, we get

\[
\begin{align*}
2(x^2 - 4x + 4) - (y^2 + 2y + 1) &= -3 + 2(4) - 1 \\
2(x - 2)^2 - (y + 1)^2 &= 4 \\
\frac{(x - 2)^2}{4} - \frac{(y + 1)^2}{4} &= 1 \quad \text{dividing by 4}
\end{align*}
\]

Hyperbola, center at \((2, -1)\).
19. \[ x^2 + 2y^2 + 6x - 4y + 9 = 0 \]
\[ x^2 + 6x + 2y^2 - 4y = -9 \]
\[ (x^2 + 6x + 9) + 2(y^2 - 2y + 1) = -9 \]
Adding \( \left[ \frac{1}{2} (6) \right]^2 = 9 \) and \( \left[ \frac{1}{2} (-2) \right]^2 = 1 \) inside the parentheses and balancing the equation, we have
\[ (x + 3)^2 + 2(y - 1)^2 = 2 \]
\[ \frac{(x + 3)^2}{2} + (y - 1)^2 = 1. \]
Ellipse, center at \((-3, 1)\) with \(a = \sqrt{2}\) and \(b = 1\).

20. \[ y^2 + 2y - 12x + 49 = 0 \]
\[ y^2 + 2y = 12x - 49 \]
Add \( \left[ \frac{1}{2} (1) \right]^2 = 1 \) to each side:
\[ y^2 + 2y + 1 = 12x - 49 + 1 = 12x - 48 \]
\[ (y + 1)^2 = 12(x - 4) \quad \text{factoring} \]
\[ (y + 1)^2 = 4(3)(x - 4) \quad p = 3 \]
Parabola, vertex at \((4, -1)\), focus at \((4 + 3, -1) = (7, -1)\).

21. \[ x^2 + 4x + 4y + 16 = 0 \]
\[ x^2 + 4x = -4y - 16 \]
We add to each side \( \left[ \frac{1}{2} \cdot 4 \right]^2 = 4: \)
\[ x^2 + 4x + 4 = -4y - 16 + 4 \]
\[ (x + 2)^2 = -4y - 12 \]
\[ (x + 2)^2 = -4(y + 3) \]
\[ (x + 2)^2 = 4(-1)(y + 3), \quad p = -1 \]
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22. \[ 3y^2 - 2x^2 - 18y - 8x + 7 = 0 \]
\[ 3y^2 - 18y - 2x^2 - 8x = -7 \]
\[ 3(y^2 - 6y) - 2(x^2 + 4x) = -7 \]
Observe that \[ \left( \frac{1}{2}(-6) \right)^2 = 9 \] and \[ \left( \frac{1}{2}(4) \right)^2 = 4 \]. Inserting these values inside the parentheses and balancing the equation, we get

\[ 3(y^2 - 6y + 9) - 2(x^2 + 4x + 4) = -7 + 3(9) - 2(4) \]
\[ 3(y - 3)^2 - 2(x + 2)^2 = 12 \]
\[ \frac{(y - 3)^2}{4} - \frac{(x + 2)^2}{6} = 1 \]

Hyperbola, center at \((-2, 3)\).

23. \[ x^2 + 2y^2 - 4x + 12y + 14 = 0 \]
\[ x^2 - 4x + 2y^2 + 12y = -14 \]
\[ (x^2 - 4x) + 2(y^2 + 6y) = -14 \] factoring
Observe that \[ \left( \frac{1}{2}(-4) \right)^2 = 4 \] and \[ \left( \frac{1}{2}(6) \right)^2 = 9 \]. Inserting these values inside the parentheses and balancing the equation, we get

\[ (x^2 - 4x + 4) + 2(y^2 + 6y + 9) = -14 + 4 + 2 \cdot 9 \]
\[ (x - 2)^2 + 2(y + 3)^2 = 8 \]
\[ \frac{(x - 2)^2}{8} + \frac{(y + 3)^2}{4} = 1 \].

Ellipse, center at \((2, -3)\).
24. \( x^2 + 4y^2 + 6x + 24y + 41 = 0 \)
\( x^2 + 6x + 4y^2 + 24y = -41 \)
\( x^2 + 6x + 4(y^2 + 6y) = -41 \)
Insert \( \left( \frac{1}{2} \right)^2 = 9 \) and balance the equation:
\[
(x^2 + 6x + 9) + 4(y^2 + 6y + 9) = -41 + 9 + 4(9)
\]
\[
(x + 3)^2 + 4(y + 3)^2 = 4
\]
\[
\frac{(x + 3)^2}{4} + \frac{(y + 3)^2}{1} = 1
\]
Ellipse, center at \((-3, -3)\) with \(a = 2\) and \(b = 1\).

25.

Distance from vertex to focus: \(3 - (-1) = 4\). Thus \(p = 4\). Since the axis is horizontal, the form of the equation is \((y - k)^2 = 4p(x - h)^2\). Thus
\[
(y - 2)^2 = 4 \cdot 4(x + 1)
\]
\[
(h, k) = (-1, 2), \ p = 4
\]
\[
(y - 2)^2 = 16(x + 1).
\]
27. y = 6

(3,3) \text{• vertex}

The distance from the vertex to the focus is 3, with the focus below the vertex; so $p = -3$.

Since the axis is vertical, the form of the equation is

$$ (x - h)^2 = 4p(y - k) $$

$$ (x - 2)^2 = 4(3)(y - 2) \quad (h, k) = (2, 2) $$

$$ (x - 2)^2 = 12(y - 2) $$

28. Form: $(y - k)^2 = 4p(x - h)$; Axis horizontal

Since the focus is 2 units to the left of the vertex, $p = -2$. Equation: $(y + 2)^2 = -8(x - 5)$.

29. (0,0) \text{• vertex}

Since the center is at $(-3, 0)$, we get for the equation

$$ \frac{(x + 3)^2}{9} + \frac{y^2}{4} = 1 \quad a = 3, \ b = 2 $$
30. Center at \((3, -2)\) [midway between the vertices]. Distance from center to a vertex is 3, so \(a = 3\). Distance from center to focus is 2, so \(c = 2\).

\[
b^2 = a^2 - c^2 = 9 - 4 = 5
\]

Form: \(\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1\) major axis vertical

Equation: \(\frac{(x-3)^2}{5} + \frac{(y+2)^2}{9} = 1\) \((h,k) = (3, -2)\)

31.

Major axis vertical: \(\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1\). From the diagram: \(a = 3\) and \(c = 1\); so \(b^2 = a^2 - c^2 = 9 - 1 = 8\); center: \((-4,1)\).

Equation: \(\frac{(x+4)^2}{8} + \frac{(y-1)^2}{9} = 1\)

32.

Major axis horizontal: \(\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1\) (form)

From the diagram: \(a = 6\) and \(b = 1\). Center: \((-1, -1)\).

Equation: \(\frac{(x+1)^2}{36} + (y+1)^2 = 1\).
33. \[ \begin{align*} \text{Distance between vertices is 8, so that } a &= 4. \text{ Center: } (-3, 1) \text{ (point midway between vertices).} \\
\text{Distance from center to one focus is 6, so that } c &= 6. \text{ The transverse axis is horizontal, resulting in the form} \\
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} &= 1. \\
\text{Since } b^2 = c^2 - a^2 = 36 - 16 = 20, \text{ we get} \\
\frac{(x + 3)^2}{16} - \frac{(y - 1)^2}{20} &= 1. \quad \text{ }(h, k) = (-3, 1) \end{align*} \]

34. \[ \begin{align*} \text{Distance from center to one focus is } 3, \text{ so } c &= 3. \text{ Conjugate axis } 4, \text{ so } b = 2. \\
\text{Form: } \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} &= 1 \quad \text{Transverse axis horizontal} \\
\text{From: } b^2 = c^2 - a^2, \text{ we have } 4 = 9 - a^2 \text{ or } a^2 = 5. \\
\text{Equation: } \frac{(x + 1)^2}{5} - \frac{(y + 2)^2}{4} &= 1 \end{align*} \]

35. Transverse axis vertical: \[ \begin{align*} \frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} &= 1 \quad \text{(form)} \\
\text{Distance from center to focus: } 8, \text{ so } c &= 8. \text{ Since } a = 6, \text{ we have } b^2 = c^2 - a^2 = 64 - 36 = 28. \\
\text{Equation: } \frac{(y + 4)^2}{36} - \frac{(x - 3)^2}{28} &= 1 \end{align*} \]

36. \[ \begin{align*} \text{Distance between vertices is 8, so that } a &= 4. \text{ Center: } (-3, 1) \text{ (point midway between vertices).} \\
\text{Distance from center to one focus is 6, so that } c &= 6. \text{ The transverse axis is horizontal, resulting in the form} \\
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} &= 1. \\
\text{Since } b^2 = c^2 - a^2 = 36 - 16 = 20, \text{ we get} \\
\frac{(x + 3)^2}{16} - \frac{(y - 1)^2}{20} &= 1. \quad \text{ }(h, k) = (-3, 1) \end{align*} \]
1.11. **Translation of Axes; Standard Equations of the Conics**

Form: \( \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 \)

Center: \((-1, -2), a = 3, c = 5\)

\[ b^2 = c^2 - a^2 = 25 - 9 = 16 \]

Equation: \( \frac{(y + 2)^2}{9} - \frac{(x + 1)^2}{16} = 1 \)

37.

\((-3, 3)\) vertex \((2, 3)\) center

\((h, k) = (2, 3); a = 2 - (-3) = 5\) (distance from center to vertex);

\(b = 2\) (length of minor axis is 4).

Form: \( \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1. \) major axis horizontal

Resulting equation: \( \frac{(x - 2)^2}{25} + \frac{(y - 3)^2}{4} = 1. \) \((h, k) = (2, 3)\)

38. Form: \((y - k)^2 = 4(p)(x - h)\) Axis horizontal

Focus: 3 units to the right of the vertex, so \(p = 3\)

Equation: \((y + 4)^2 = 4(3)(x - 5)\) or \((y + 4)^2 = 12(x - 5)\)

39. Form: \( \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1. \)

Distance from center to vertex is 2, so that \(a = 2.\) From \(x - 2y = 1, y = \frac{1}{2}x - \frac{1}{2}.\) So the slope \(m\) of one of the asymptotes is \(\frac{1}{2}.\) But \(m = \frac{b}{a}.\) Thus \(\frac{1}{2} = \frac{b}{a} = \frac{a}{2}\) or \(b = 1.\) The equation is

\[ \frac{(x - 1)^2}{4} - \frac{y^2}{1} = 1. \) \((h, k) = (1, 0)\)

40. Form: \((x + 1)^2 = 4(p)(y - 3)\) Axis vertical

The coordinates of the origin \((0, 0)\) satisfy the equation:

\((0 + 1)^2 = 4(p)(0 - 3)\) or \(4p = -\frac{1}{3}\)

Equation: \((x + 1)^2 = -\frac{1}{3}(y - 3)\)

The form \((y - 3)^2 = 4p(x + 1)\) Axis horizontal

leads to \((y - 3)^2 = 9(x + 1)\)

41.

\((-3, 6)\) vertex \((-3, -3)\) center

\(a = 5\)

\((-3, 1)\) center

\(c = 4\)

\[ b^2 = a^2 - c^2 = 25 - 16 = 9 \]
\[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad \text{major axis vertical}
\]
\[
\frac{(x+3)^2}{9} + \frac{(y-1)^2}{25} = 1
\]

42.

Since \((4,0)\) is the center, we can read off \(c = 4\) and \(b = 3\).

From \(b^2 = c^2 - a^2\), we get \(9 = a^2 - 16\) or \(a^2 = 25\).

Form: \[\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1\] Major axis horizontal

Equation: \[\frac{(x-4)^2}{25} + \frac{y^2}{9} = 1\] \((h,k) = (4,0)\)

43. Distance from vertex to focus: \(4 - 1 = 3\). Since the focus is to the left of the vertex, \(p = -3\).

Form:
\[
(y-k)^2 = 4p(x-h) \quad \text{axis horizontal}
\]
\[
(y-k)^2 = -12(x-h). \quad \text{ } p = -3
\]
Equation: \((y+2)^2 = -12(x-4)\). \((h,k) = (4,-2)\)

44.

Distance from center to one focus is 4, so \(c = 4\)

Distance from center to one vertex is 3, so \(a = 3\)

Also \(b^2 = c^2 - a^2 = 16 - 9 = 7\)

Form: \[\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1\] Transverse axis horizontal

Equation: \[\frac{(x-3)^2}{9} - \frac{(y-1)^2}{7} = 1\]

45. Since the vertex is midway between the focus and directrix, its coordinates are \((-2,-4)\). Since \(p = -4\), we have
\[
(x+2)^2 = 4\cdot(-4)(y+4)
\]
\[
(x+2)^2 = -16(y+4).
\]
46. Distance from center to one focus is 3, so \( c = 3 \)
Major axis is 8, so \( a = 4 \)
Also, \( b^2 = a^2 - c^2 = 16 - 9 = 7 \)
Form: \[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad \text{Major axis vertical}
\]
Equation: \[
\frac{(x-1)^2}{7} + \frac{(y-1)^2}{16} = 1
\]

47. \((h,k) = (-1,1); a = 3 - 1 = 2 \) (distance from center to vertex);
\( c = 1 - (-2) = 3 \) (distance from center to focus); \( b^2 = c^2 - a^2 = 9 - 4 = 5 \).
Form: \[
\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1. \quad \text{transverse axis vertical}
\]
Equation: \[
\frac{(y-1)^2}{4} - \frac{(x+1)^2}{5} = 1. \quad (h,k) = (-1,1)
\]

48. \[y = 0.1x(20 - x)\]
\[y = 2x - 0.1x^2\]
\[10y = 20x - x^2 \] (multiply by 10)
\[x^2 - 20x = -10y\]
\[x^2 - 20x + 100 = -10y + 100 \] add \( \left[\frac{1}{2}(-20)\right]^2 \) to each side
\[(x - 10)^2 = -10(y - 10)\]
Vertex: \((10,10)\); Maximum height: 10 units.

49. Multiply both sides of the given equation by \( \frac{81}{x} \) and then multiply out the right side:
\[\frac{81}{x}y = -x^2 + 16x + 17\]
\[x^2 - 16x = -\frac{81}{x}y + 17\]
\[x^2 - 16x + 64 = -\frac{81}{x}y + 17 + 64 \] add \( \left[\frac{1}{2}(-16)\right]^2 \) to each side
\[(x-8)^2 = -\frac{81}{x}(y - 5)\]
Vertex: \((8,5)\); maximum height: 5 units.
Chapter 1 Review

1. Slope of line segment joining (3, 10) and (7, 4): $\frac{-3}{2}$.
   Slope of line segment joining (4, 2) and (7, 4): $\frac{2}{3}$.
   Since the slopes are negative reciprocals, the line segments are perpendicular.

2. Midpoint of line segment: \[
\left(\frac{1}{2} (-1 + 5), \frac{1}{2} (-2 + 4)\right) = (2,1)
\]
   Slope of line segment: \[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{5 - (-1)} = \frac{6}{6} = 1
\]
   Slope of perpendicular: $-1$
   Equation of bisector: \[
y - 1 = (-1)(x - 2) \text{ or } x + y = 3
\]

3. $C = \frac{5}{9}(F - 32)$
   By assumption $C = F$,
   \[
   F = \frac{5}{9}(F - 32)
   F = \frac{5}{9}F - \frac{160}{9}
   F - \frac{5}{9}F = -\frac{160}{9}
   \frac{4}{9}F = -\frac{160}{9}
   F = -40^{\circ}.
\]

4. Freezing point of water

5. Slope of line segment joining (−1, 5) and (3, 9): 1.
   Slope of line segment joining (3, 1) and (7, 5): 1.
   Slope of line segment joining (−1, 5) and (3, 1): −1.
   Slope of line segment joining (3, 9) and (7, 5): −1.
   Since opposite sides are parallel, the figure is a parallelogram. Moreover, since the line segment joining (−1, 5) and (3, 1) is perpendicular to the line segment joining (−1, 5) and (3, 9), the figure must be a rectangle. Finally:
   Length of line segment joining (−1, 5) and (3, 1) = $4\sqrt{2}$.
   Length of line segment joining (−1, 5) and (3, 9) = $4\sqrt{2}$.
   Thus the figure is a square.

6. Solving $x + 2y - 5 = 0$ for $y$, we have $y = -\frac{1}{2}x + \frac{5}{2}$, whose slope is $-\frac{1}{2}$.
   Slope of perpendicular: 2
   Equation of the line: $y - 1 = 2(x - 4) \text{ or } 2x - y - 7 = 0$

7. $3x + y = 3$
   \[
y = -3x + 3 \quad y = mx + b
   \]
   Since $m = -3$, we get
   \[
y - 5 = -3(x + 1) \quad y - y_1 = m(x - x_1)
   3x + y - 2 = 0.
\]

8. Writing the lines in slope-intercept form:
   \[
y = 4x - 7, \quad y = -\frac{1}{4}x + \frac{1}{4}, \quad y = -2x
   \]
   The first two lines are perpendicular (and none are parallel).

9. $x^2 = (1 - 0)^2 + (-2 - 0)^2 = 1 + 4 = 5$. We now get
   \[
   (x - 1)^2 + (y + 2)^2 = 5 \text{ or } x^2 + y^2 - 2x + 4y = 0.
\]
10. \( r = \text{distance from the center to the } x\text{-axis} = 5 \)
\((x - 2)^2 + (y - 5)^2 = 5^2 \) or \( x^2 + y^2 - 4x - 10y + 4 = 0 \)

11. 
\[
\begin{align*}
x^2 + y^2 + 2x + 2y &= 0 \\
x^2 + 2x + y^2 + 2y &= 0 \\
(x^2 + 2x + 1) + (y^2 + 2y + 1) &= 1 + 1 \\
(x + 1)^2 + (y + 1)^2 &= 2
\end{align*}
\]
Center: \((-1, -1); \quad r = \sqrt{2}\). 

12. Write the equation in standard form:
\[
\begin{align*}
x^2 + y^2 - 10x - 8y + 16 &= 0 \\
x^2 - 10x + y^2 - 8y &= -16
\end{align*}
\]
Adding \(\left[\frac{1}{2}(-10)\right]^2 = 25\) and \(\left[\frac{1}{2}(-8)\right]^2 = 16\) to each side we get
\[
\begin{align*}
x^2 - 10x + 25 + y^2 - 8y + 16 &= -16 + 25 + 16 \\
(x - 5)^2 + (y - 4)^2 &= 25
\end{align*}
\]
Center: \((5, 4)\), radius: 5. So the center is 5 units from the \(y\)-axis.

13. Ellipse, major axis vertical, \(a = 4, \quad b = 3\). From \(b^2 = a^2 - c^2\),
\[
\begin{align*}
9 &= 16 - c^2 \\
c &= \pm \sqrt{7}.
\end{align*}
\]
Vertices: \((0, \pm 4)\); foci: \((0, \pm \sqrt{7})\). (See sketch in answer section of book.)

14. \(x^2 + 4y^2 = 1\) or \(x^2 + \frac{y^2}{1/4} = 1\)

Ellipse, \(a = 1, \quad b = \frac{1}{2}\), major axis horizontal
\[
\begin{align*}
b^2 &= a^2 - c^2 \\
\frac{1}{4} &= 1 - c^2 \\
c &= \pm \frac{\sqrt{3}}{2}
\end{align*}
\]
Vertices: \((\pm 1, 0)\), foci: \(\left(\pm \frac{\sqrt{3}}{2}, 0\right)\)

15. \(\frac{y^2}{4} - \frac{x^2}{7} = 1\)

Hyperbola, transverse axis vertical with \(a = 2\) and \(b = \sqrt{7}\). From \(b^2 = c^2 - a^2, \quad 7 = c^2 - 4\) and \(c = \pm \sqrt{11}\). So the vertices are at \((0, \pm 2)\) and the foci are at \((0, \pm \sqrt{11})\). Using \(a = 2\) and \(b = \sqrt{7}\), we draw the auxiliary rectangle and sketch the curve.
16. \( \frac{x^2}{9} - \frac{y^2}{16} = 1 \)

Hyperbola, \( a = 3, b = 4 \), transverse axis horizontal

\[
\begin{align*}
    b^2 &= c^2 - a^2 \\
    16 &= c^2 - 9 \\
    c &= \pm 5
\end{align*}
\]

Vertices: \((\pm 3, 0)\), foci: \((\pm 5, 0)\)

Use \( a = 3 \), and \( b = 4 \), to draw the auxiliary rectangle.

17. Parabola, axis horizontal. From \( y^2 = -3x \), we have

\[
y^2 = 4 \left( -\frac{3}{4} \right) x.
\]

Inserting 4

Thus, \( p = -\frac{3}{4} \), placing the focus at \( \left( -\frac{3}{4}, 0 \right) \). (See sketch in answer section of book.)

18. \( x^2 = 9y \) or \( x^2 = 4 \left( \frac{9}{4} \right) y \)

Parabola, \( p = \frac{9}{4} \), so that the focus is at \( \left( 0, \frac{9}{4} \right) \) and the directrix is \( y = -\frac{9}{4} \).

19. \( y^2 + 6y + 4x + 1 = 0 \)

\[
\begin{align*}
    y^2 + 6y &= -4x - 1 \\
    \left( y + \frac{3}{2} \right)^2 &= -4x - 1 + 9 \\
    \left( y + \frac{3}{2} \right)^2 &= -4x + 8 \\
    \left( y + \frac{3}{2} \right)^2 &= 4(-1)(x - 2) \\
    p &= -1
\end{align*}
\]
Parabola, vertex at (2, -3), focus at (2 - 1, -3) = (1, -3).

20. \( x^2 + y^2 - 8x + 10y - 4 = 0 \)
\[ x^2 - 8x + y^2 + 10y = 4 \]
Add \( \left[ \frac{1}{2}(-8) \right]^2 = 16 \) and \( \left[ \frac{1}{2}(10) \right]^2 = 25 \) to each side:
\[ x^2 - 8x + 16 + y^2 + 10y + 25 = 4 + 16 + 25 \]
\[ (x - 4)^2 + (y + 5)^2 = 45 = 9 \cdot 5 \]

Circle with center at (4, -5) and radius 3\( \sqrt{5} \).

21. \( 16x^2 - 64x + 9y^2 + 18y = 71 \)
\[ 16(x^2 - 4x) + 9(y^2 + 2y) = 71 \quad \text{factoring 16 and 9} \]
Note that \( \left[ \frac{1}{2}(-4) \right]^2 = 4 \) and \( \left[ \frac{1}{2}(2) \right]^2 = 1 \). Inserting these values inside the parentheses and balancing the equation, we get
\[ 16(x^2 - 4x + 4) + 9(y^2 + 2y + 1) = 71 + 16 \cdot 4 + 9 \cdot 1 \]
\[ 16(x - 2)^2 + 9(y + 1)^2 = 144 \]
\[ \frac{(x - 2)^2}{16} + \frac{(y + 1)^2}{16} = 1. \quad \text{dividing by 144} \]

Ellipse, center at (2, -1), major axis vertical, \( a = 4, \ b = 3 \).

22. \( x^2 + 4x + 8y - 20 = 0 \)
\[ x^2 + 4x = -8y + 20 \]
\[ x^2 + 4x + 4 = -8y + 20 + 4 \quad \left[ \frac{1}{2}(4) \right]^2 = 4 \]
\[ (x + 2)^2 = -8(y - 3) \]
\[ (x + 2)^2 = 4(-2)(y - 3) \quad \text{axis vertical, vertex at (-2, 3)} \]
Since \( p = -2 \), the focus is at \( (-2, 3 - 2) = (-2, 1) \). (See graph in Answer Section.)
23. \( x^2 - y^2 - 4x + 8y - 21 = 0 \)
\( x^2 - 4x - y^2 + 8y = 21 \)
\((x^2 - 4x) - (y^2 - 8y) = 21\)
Adding \( \left[ \frac{1}{2}(-4) \right]^2 = 4 \) and \( \left[ \frac{1}{2}(-8) \right]^2 = 16 \) inside the parentheses and balancing the equation we get
\( (x^2 - 4x + 4) - (y^2 - 8y + 16) = 21 + 4 - 1(16) \)
\( \frac{(x - 2)^2}{9} - \frac{(y - 4)^2}{9} = 1. \)
Hyberbola, center at \((2, 4)\).

24. Since the directrix is \( x = -2 \), the focus is at \((2, 0)\). Form: \( y^2 = 4px \) or \( y^2 = 8x \).

25. Form:
\( (x - h)^2 = 4p(y - k). \)  
axis vertical
Distance from vertex \((1, 3)\) to directrix \( y = 0 \) is 3, so that \( p = 3 \). The equation is
\( (x - 1)^2 = 4(3)(y - 3) \)  \( (h, k) = (1, 3), \ p = 3 \)
or
\( (x - 1)^2 = 12(y - 3). \)

26.

Distance from center to one vertex is 4, so \( a = 4 \)
Distance from center to a focus is 2, so \( c = 2 \)
Also, \( b^2 = a^2 - c^2 = 16 - 4 = 12 \)
Form: \( \frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1 \)  Major axis vertical
Equation: \( \frac{(x + 2)^2}{12} + \frac{(y + 4)^2}{16} = 1 \)

27. Form: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \ a = 4; \ c = 3; \ b^2 = a^2 - c^2 = 16 - 9 = 7. \)
Equation: \( \frac{x^2}{4^2} + \frac{y^2}{7^2} = 1. \)
28. Since the vertex is at (3,0), \( a = 3 \). By Eq. (1.21)

\[
y = \pm \frac{b}{a} x = \pm \frac{3}{4} x \quad \text{or} \quad \frac{b}{a} = \frac{3}{4}
\]

Since \( a = 3 \), \( b = \frac{9}{4} \). Form: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) Transverse axis horizontal;

Equation: \( \frac{x^2}{9} - \frac{y^2}{(9/4)^2} = 1 \) or \( 9x^2 - 16y^2 = 81 \)

29. 

\[
\begin{array}{c}
\text{y} \\
(0,5) \quad \text{vertex} \\
(0,2) \quad \text{center} \\
(0,-1) \quad \text{vertex} \\
(0,-2) \quad \text{focus}
\end{array}
\]

Center: \((0, 2)\) (midway between vertices). Distance from center to vertex is 3, so that \( a = 3 \). Distance from center to focus is 4, so that \( c = 4 \). Thus \( b^2 = c^2 - a^2 = 16 - 9 = 7 \). Since the transverse axis is vertical, the form is

\[
\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1,
\]

and the equation is

\[
\frac{(y - 2)^2}{9} - \frac{x^2}{7} = 1. \quad (h, k) = (0, 2)
\]

30. Form: \( (y - k)^2 = 4(p)(x - h) \)

Vertex: \((-4, 2)\) [midway between focus and directrix]

Since \( p = -4 \), the equation is \( (y - 2)^2 = -16(x + 4) \)

31. 

\[
\begin{array}{c}
\text{y} \\
(0,3) \quad \text{focus} \\
\text{vertex} \\
\hline
y = 1 \\
0 \\
x
\end{array}
\]

Since the vertex is midway between the focus and directrix, its coordinates are \((0, 2)\). It follows that \( p = +1; \) so the equation is

\[
(x - 0)^2 = 4(1)(y - 2) \quad \text{or} \quad x^2 = 4(y - 2).
\]
Chapter 1. Introduction to Analytic Geometry

32.

Distance from center to a vertex is 2, so $a = 2$
Distance from center to a focus is 3, so $c = 3$
Also, $b^2 = c^2 - a^2 = 9 - 4 = 5$
Form: \[
\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1 \quad \text{Transverse axis vertical}
\]
Equation: \[
\frac{(y + 1)^2}{4} - \frac{(x - 2)^2}{5} = 1
\]

33.

$c = 2$ (distance from center to focus); $a = 4$ (distance from center to vertex);
$b^2 = a^2 - c^2 = 16 - 4 = 12$. Form: \[
\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1. \quad \text{major axis vertical}
\]
Equation: \[
\frac{(x - 4)^2}{12} + \frac{(y + 1)^2}{16} = 1. \quad (h, k) = (4, -1)
\]

34. See graph in Answer Section.

35. $y = (x + 1)^3$
   
   \underline{Intercepts}. If $x = 0$, then $y = 1$. If $y = 0$, then $x = -1$.
   
   \underline{Symmetry}. None: replacing $x$ by $-x$ or $y$ by $-y$ changes the equation.
   
   \underline{Asymptotes}. None: the equation is not in the form of a fraction with $x$ in the denominator.
   
   \underline{Extent}. All $x$ and all $y$.
   
   \underline{Graph}.

36. See graph in Answer Section.
37. **Intercepts**. If \( x = 0 \), then \( y = 0 \); if \( y = 0 \), then \( x(x - 4) = 0 \) and thus \( x = 0, 4 \).

**Symmetry.** Replacing \( y \) by \(-y\), we get \((-y)^2 = x(x - 4)\), which reduces to the given equation. So the curve is symmetric with respect to the \( x\)-axis.

**Asymptotes.** None (equation is not in the form of a fraction).

**Extent.** Solving for \( y \) we have
\[
y = \pm \sqrt{x(x - 4)}.
\]

If \( x > 4 \), \( x(x - 4) > 0 \). If \( 0 < x < 4 \), \( x(x - 4) < 0 \) [for example, if \( x = 2 \), we get \( 2(2 - 4) = -4 \)].

If \( x < 0 \), \( x(x - 4) > 0 \), since both factors are negative. So the extent is \( x \leq 0 \) and \( x \geq 4 \).

**Graph.**

![Graph of \( y = \pm \sqrt{x(x - 4)} \)](image)

38. See graph in Answer Section.

39. \[ y = \frac{x}{x^2 - 4} \]

**Intercept.** \((0, 0)\)

**Symmetry.** Replacing \( x \) by \(-x\) and \( y \) by \(-y\) we get
\[
-y = \frac{-x}{(-x)^2 - 4}
\]

which reduces to
\[
y = \frac{x}{x^2 - 4}.
\]

The graph is therefore symmetric with respect to the origin.

**Asymptotes.** Vertical: setting the denominator equal to 0 results in
\[
x^2 - 4 = 0 \quad \text{and} \quad x = \pm 2.
\]

If \( x \) gets large, \( y \) approaches 0, so that the \( x\)-axis is a horizontal asymptote.

**Extent.** All \( x \) except \( x = 2 \) and \( x = -2 \).

**Graph.**

![Graph of \( y = \frac{x}{x^2 - 4} \)](image)

40. See graph in Answer Section.
41. Placing the vertex at the origin, one point on the parabola is (0.90, 0.60), as shown in the figure. The form is \( y^2 = 4px \). To find \( p \), we substitute the coordinates of the point in the equation:

\[
(0.60)^2 = 4p(0.90)
\]

\[
p = \frac{(0.60)^2}{(4)(0.90)} = 0.10.
\]

\[(0.90,0.60)\]

To be at the focus, the light must be placed 0.10 feet from the vertex.

42. \[
\frac{x^2}{25.0^2} + \frac{y^2}{15.0^2} = 1 \quad \text{where} \quad a = 25.0, \ b = 15.0
\]

Let \( x = 15.0 \) and solve for \( y \):

\[
\frac{15.0^2}{25.0^2} + \frac{y^2}{15.0^2} = 1 \quad \text{and} \quad y = 12.0 \text{ft}
\]

43. \[
V = (6 - 2x)(6 - 2x)x = x(6 - 2x)^2
\]

Volume=length\times width\times height.

See graph in answer section of book.

44. See graph in answer section of book.

45. See Exercise 41, Section 1.9:

\[
e = \frac{A - P}{A + P};
\]

\[
P = 4000 + 119 = 4119 \text{ mi}; \ A = 4000 + 122,000 = 126,000 \text{ mi};
\]

\[
e = \frac{126,000 - 4119}{126,000 + 4119} = 0.94.
\]